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THE OPTIMUM STRATEGY IN BLACKJACK

ROGER R. BALDWIN, WILBERT E. CANTEY, HERBERT MAISEL, JAMES P. MCDERMOTT*

Aberdeen Proving Ground, Maryland

This article discusses the card game blackjack as played in the casinos of Las Vegas. The basic rules for the game are described in detail. The player's strategic problems are analyzed with the objective of finding the strategy maximizing his mathematical expectation.

A mathematical expression is derived giving a general solution to the player's problem of standing pat with a given hand versus drawing additional cards. No general solutions are possible for the other major strategic problems, however, and a detailed examination of individual situations is required. The formulas and methods for the case analysis are stated, but computational details are omitted. Similarly, the formula for the player's mathematical expectation is stated, but its numerical evaluation is not described. Detailed discussion is given to the problems arising in the combinatorial type of computations required by blackjack.

The "optimum strategy" determined by the above analysis differs substantially from the published strategies of card experts and the usual style of play in the casinos.

I. THE GAME OF BLACKJACK

BLACKJACK or twenty-one is one of the most widely played games in American homes and clubs and traditionally rivals poker for popularity in the Armed Forces. In Las Vegas, Reno, and other parts of the wide-open spaces blackjack ranks with poker, roulette, and craps as one of the four standard gambling games. Of these four, however, blackjack is by far the most neglected in the scientific literature of gambling and offers a relatively unexplored area for mathematical and statistical analysis.

It should be made clear at the outset that this paper deals exclusively with the "house" game of blackjack and not the "private" game. In the house game a representative of the gambling casino is permanent dealer, and his strategy is completely fixed by known house rules. The fixed and known nature of the dealer's strategy is vital in reducing the mathematical and computational problems in analyzing blackjack to manageable proportions.

Each gambling casino has a set of blackjack rules which agree with those of other casinos on the main points but which usually differ on details. Therefore, in selecting a variation of the game of blackjack for analysis, the best that could be done was to consider rules which are common but not universal. A presentation of these rules follows.

* The authors were ably assisted by Otto Dyaktra, General Foods Laboratories, Hoboken, N. J.

1. *The Number of Players.* A dealer and from one to six players.

2. *The Pack.* An ordinary 52-card deck.

3. *Betting.* The players make their bets *before* any cards are dealt. The house establishes a minimum and a maximum bet.

4. *The Deal.* The players and dealer each receive two cards. Each player gets both cards face down. The dealer receives *one card face up* and one card face down. Cards received face down in the deal or draw are commonly known as "hole cards."

5. *The Numerical Value of the Cards.* The numerical value of an ace is 1 or 11 as the player chooses, the numerical value of a face card is 10, and the numerical value of all other cards is simply their face value. The numerical value or total of a hand is the sum of the numerical values of the cards in the hand.

6. *Object of the Player.* To obtain a total which is greater than the dealer's but does not exceed 21.

7. *Naturals.* An ace and a face card or ten dealt on the first two cards to either player or dealer constitutes a "natural" or "blackjack." If a player has a natural and the dealer does not, the player receives $1\frac{1}{2}$ times his original bet from the dealer. If a player does not have a natural and the dealer does, the player loses his original bet. If both player and dealer have naturals, no money changes hands.

8. *The Draw.* A player is not required to increase the number of cards in his hand and may look at his hole cards and elect to "stand." Otherwise, he may require that the dealer give him additional cards, face up, one at a time. If the player goes over 21 ("busts"), he immediately turns up his hole cards and pays his bet to the dealer. After each player has drawn his cards, starting with the player at the dealer's left and proceeding in a clockwise fashion, the dealer turns up his hole card. If his total is *16 or less*, he must draw a card and continue to draw cards until his total is *17 or more*, at which point he must stand. If the dealer has an ace, and counting it as 11 would bring his total to 17 or more without exceeding 21, he must count the ace as 11 and stand.

9. *The Settlement.* If the player does not go over 21 ("bust") and the dealer does, the player wins an amount equal to his original bet. If neither player nor dealer busts, the person with the higher total wins an amount equal to the player's original bet. If neither player nor dealer busts and both have the same total, no money changes hands.

10. *Splitting Pairs.* In the following rules a pair is defined as two cards which are identical except for suit, such as two jacks, two aces, or two tens. If the player's hole cards form a pair, he may choose to turn them face up and treat them as the initial cards in two separate "twin" hands. This strategy is known as "splitting pairs." The original bet goes on one of the split cards, and an equal amount is bet on the other card. The player automatically receives a second card face down on each of the split cards and may continue drawing cards face up to both twin hands as long as he desires. An exception to this rule is made in the case of split aces where the player may draw only *one* more card to each ace. Furthermore, if a face card or ten falls on one of the split aces, the hand is not counted as a natural but as ordinary 21. (Similarly, the player splitting a pair of face cards or tens who draws an ace holds an ordinary

21.) Finally, if a player splits a pair and receives a third card of the same type, he is not permitted to undertake further splitting.

11. *Doubling Down.* After looking at his hole cards a player may elect to double his bet and draw one and only one more card. This strategy is known as "doubling down." A player who elects to double down turns up his hole cards and receives his third card face down. A player splitting any pair except aces, after receiving an additional card on each of the split cards, may elect to double down on one or both of his twin hands.

The reader who is primarily interested in card playing results should skip to Section VI for a description of the optimum strategy. Section II gives the derivation of the basic mathematical formulas; Section III discusses the probability the dealer obtains various final totals; Section IV describes the methods of analysis for special situations—soft hands, doubling down, and splitting pairs; and Section V presents the method of calculating the player's mathematical expectation.

II. THE BASIC "DECISION EQUATION"

Consider two types of hands held by the player: the type of hand where the player's total x has one unique value not exceeding 21, and the type of hand consisting of one or more aces in such a way that the player's total has two values not exceeding 21. (In this situation x is defined to equal the larger total.) The second type of hand is known in gambling terminology as a "soft" hand or "soft" total and requires a separate strategy.

Some notation must be defined. Let D be the numerical value of the dealer's up card. $D=2, 3, \cdots, 10, (1, 11)$. Let $M(D)$ be an integer such that if the dealer's up card is D and the player's total x is unique and less than $M(D)$, the player should draw; while if x (unique) $\geqq M(D)$, the player should stand. The set of integers $M(D)$ are known as the minimum standing numbers for unique hands. Let us define $M^*(D)$ in the same way for soft hands.

The assumption is made that a good strategy for drawing may be defined by the set of minimum standing numbers $M(D)$ and $M^*(D)$. In other words, if it is good strategy for a player to stand on a given total, it is assumed to be good strategy for him to stand on all higher totals. This assumption is almost always correct and, in particular, holds true when player and dealer draw from a full deck. The assumption breaks down in certain "pathological" cases, however, when draws are made from a severely depleted deck with an unusual assortment of cards remaining.

The first step is to compare the mathematical expectation of player 1 using $M(D)=x$ with that of player 2 using $M(D)=x+1$ where x takes on integral values not exceeding 21. The comparison is $M^*(D)=x$ versus $M^*(D)=x+1$ in the case of soft hands. Players 1 and 2 employ the same strategy except when their total is x. Player 1 stands in this situation, while in the case of unique x player 2 draws exactly one more card. Thus a comparison of the mathematical expectation of players 1 and 2 for unique hands is equivalent to comparing $E_{s,x}$, the expectations of a player standing on a total of x, with $E_{d,x}$, the expectation of a player with a total of x who draws *exactly one* card. In the case of soft hands player 1 stands, while player 2 draws *one or more* cards. For example,

if player 2 draws a five to soft 17, obtaining a total of unique 12, in most cases he should draw again. Thus the comparison of $E_{s,x}$ and $E_{d,x}$ in the case of soft hands is *not* equivalent to a comparison of the mathematical expectations of players 1 and 2. We shall use the result, however, that if $E_{d,x} > E_{s,x}$, the player should draw to soft x.

When, for unique x, $E_{d,x} - E_{s,x}$ is a non-increasing function of x, $M(D)$ is easily obtained as the smallest integral value of x for which $E_{d,x} - E_{s,x} < 0$. The function is non-increasing in almost all cases, including the case of drawing from a full deck, and increases with x only in certain of those "pathological" situations previously discussed.

The following derivation of $E_{d,x} - E_{s,x}$ holds for both unique and soft hands. Let us define T, a random variable, as the final total obtained by the dealer. If $T > 21$ or if $T < x$, the player standing on x wins the bet, assumed here to be one unit. If $T = x$, no money changes hands, while if $x < T \leqq 21$, the player loses one unit. Consequently,

$$\begin{aligned} E_{s,x} &= P(T > 21) + P(T < x) - P(x < T \leqq 21) \\ &= 2P(T > 21) - 1 + 2P(T < x) + P(T = x). \end{aligned}$$

In discussing $E_{d,x}$ one must define a second random variable, J, as the total obtained by the player upon drawing one card. In cases where this total can take on two values not exceeding 21, J represents the larger total.

$T \geqq 17$, so if $J < 17$, the player wins when $T > 21$ and loses for all other values of T. His mathematical expectation in this situation is

$$P(T > 21) - [1 - P(T > 21)] = 2P(T > 21) - 1.$$

If $17 \leqq J \leqq 21$, the player's mathematical expectation is

$$P(T > 21) + P(T < J) - P(J < T \leqq 21).$$

If $J > 21$, the player's mathematical expectation is -1.

The value of J affects T only through eliminating the possibility that the dealer draws one particular card of value $J - x$. Consequently, little error is introduced in making the assumption that J and T are independent and writing

$$\begin{aligned} E_{d,x} = {} & P(J < 17)[2P(T > 21) - 1] - P(J) > 21) \\ & + \sum_{j=17}^{21} P(J = j)[P(T > 21) + P(T < j) - P(j < T \leqq 21)]. \end{aligned}$$

Subtracting off $E_{s,x}$ and performing some straightforward algebraic manipulation

$$\begin{aligned} E_{d,x} - E_{s,x} = {} & -2P(T < x) - P(T = x) - 2P(T > 21)P(J > 21) \\ & + 2P(T < J \leqq 21) + P(T = J \leqq 21). \end{aligned}$$

This is the most general form of the decision equation.

Since $T \geqq 17$, the first two terms are zero for $x < 17$. $P(J > 21)$ is also zero for x unique and less than 12 and for all values of soft x. Consequently,

$E_{d,x}-E_{s,x}\geqq 0$ for x(unique)<12 and x(soft)<17, and, therefore, $M(D)>11$ and $M^*(D)>16$ for all D.

Let us consider next the evaluation of the decision equation when $12\leqq x$ (unique)$\leqq 16$. The first two terms are zero, and the last term may be rewritten using the independence of J and I.

$$E_{d,x}-E_{s,x}=-2P(T>21)P(J>21)+\sum_{t=17}^{21}P(T=t)\left[2P(t<J\leqq 21)+P(J=t)\right].$$

An assumption is introduced at this point that the probability distribution of $J-x$, the single card drawn by the player, is given by $P(J-x=10)=4/13$ and $P(J-x=i)=1/13$ $i=2, 3, \cdots, 9, (1, 11)$, i.e., each card in the deck has an equal chance of being drawn by the player. This assumption may be incorrect in individual hands but holds true in the sample space composed of all 52! permutations of the deck. With this assumption

$$P(J>21)=1/13(x-8) \text{ for } x\text{(unique)}\geqq 12, \text{ and}$$

$$P(t<J\leqq 21)=1/13(21-t), \qquad P(J=t)=1/13 \text{ for } 17\leqq t\leqq 21. \text{ Then}$$

$$E_{d,x}-E_{s,x}=-2/13(x-8)P(T>21)+\sum_{t=17}^{21}1/13(43-2t)P(T=t).$$

It is not necessary to compute $E_{d,x}-E_{s,x}$ for all values of $12\leqq x\leqq 16$. One may set $E_{d,x}-E_{s,x}=0$, and, since the function decreases linearly with x, obtain a single solution, $x=x_0$.

$$x_0=8+\frac{\sum_{t=17}^{21}(21\tfrac{1}{2}-t)P(T=t)}{P(T>21)}.$$

Then if $x_0<12$, $M(D)=12$; if $x_0>16$, $M(D)>16$; and if $12\leqq x_0\leqq 16$, $M(D)=[x_0]+1$ where, in general, $[z]$ is defined to be the largest integer not greater than z. As one might expect, the greater the probability that the dealer busts, the lower the player's minimum standing number. Less obvious is the result that for a given value of $P(T>21)$ the greater the dealer's chances for a good hand, the *lower* the player's minimum standing number. For example, if $P(T>21)=2/5$ and $P(T=18)=3/5$, $M(D)=14$; while if $P(T>21)=2/5$ and $P(T=19)=3/5$, $M(D)=12$.

In the case where x(unique)$=17$

$$E_{d,17}-E_{s,17}=-18/13P(T>21)-5/13P(T=17)+\sum_{t=18}^{21}1/13(43-2t)P(T=t).$$

Since an evaluation of $P(T=t)$ will show that $E_{d,17}-E_{s,17}<0$ for all D, and consequently $M(D)\leqq 17$, it is not necessary to make any further evaluations of the decision equation for unique hands.

In the case of soft hands the only remaining use for the decision equation arises where x(soft)$=17$. In that situation

$$E_{d,17} - E_{s,17} = -1/13P(T = 17) + \sum_{t=18}^{21} 1/13(43 - 2t)P(T = t)$$

which an evaluation of $P(T=t)$ will show is positive for all D, proving that $M^*(D)>17$.

III. EVALUATION OF THE DEALER'S PROBABILITIES, $P(T=t)$.

Calculation of x_0 and the above expressions for $E_{d,17}-E_{s,17}$ for unique and soft 17 clearly requires only an evaluation of the dealer's probabilities, a task accomplished in three stages. In the first stage an exact evaluation was made of $P(T_3=v)$, the probability that the dealer obtains a total of v on his first three cards. These numbers, known as "three-card probabilities," were computed separately for each value of D, the dealer's first card. In cases where the rules required the dealer to stand on two cards, the probabilities for the totals thus obtained were included with the three-card probabilities.

In the second stage a table was developed which gives approximate values for $P(T=t/T_p=t_p)$, the conditional probability that the dealer obtains a final total t ($t\geqq 17$) given a partial total t_p ($t_p<17$). The table was developed under the following simplifying assumptions: (1) the probability of drawing any card in the deck is 1/52 ("equiprobability"); and (2) no matter how many cards the player draws, the probability of receiving any particular card on the next draw is still 1/52 ("sampling *with* replacement").

In the third stage the results from the previous stages are combined yielding the following approximation for $P(T=t)$ for $t\geqq 17$.

$$P(T = t) = P(T_3 = t) + \sum_{j<17} P(T_3 = j)P(T = t/T_p = j).$$

One may wonder about the accuracy of the approximate values for $P(T=t)$. Unfortunately, calculation of exact values is an exceedingly laborious task which has been completed in only two cases, $D=6$ and 10. The comparison of the exact and approximate probabilities for $D=6$ shows considerably greater error in the approximate probabilities than in the case $D=10$. Furthermore, some complicated heuristic arguments not presented here indicate that the error for any D will not be appreciably greater than for the case $D=6$ given below.

t	17	18	19	20	21	>21
$P(T=t)$ Exact	.166948	.106454	.107192	.100705	.097878	.420824
$P(T=t)$ Approx.	.167625	.107234	.108017	.101260	.098364	.417499

The error in each approximate probability in the above table is less than 1% of the corresponding exact probability. Assuming that the errors are less than 1% for all D, an examination of the expression for x_0 previously derived indicates that the maximum possible error in x_0 is 2% of the second term. A study of the results for x_0 shows that $D=10$ is the only case where such an error might affect $M(D)$. In this case $x_0=16.01$ when calculated with approximate probabilities and 15.97 with exact. Since these values bracket 16.00, a more detailed analysis was made which in considering the probability distribution of $J-x$ assumed that one ten-counting card was withdrawn from the full deck. With this

more realistic assumption $x_0 > 16$ and $M(10) = 17$. It is easily shown, furthermore, that a maximum error of 1% in each value of $P(T=t)$ cannot possibly affect the conclusions that $M(D) \leqq 17$ and $M^*(D) > 17$.

A table of approximate dealer's probabilities, $P(T=t)$, is given below for all values of t and D.

		t						
		17	18	19	20	21	natural 21	>21
	2	.141781	.134885	.131432	.123829	.119581	.000000	.348492
	3	.133533	.133052	.126197	.122563	.114903	.000000	.369751
	4	.132206	.116037	.122553	.117930	.114292	.000000	.396983
D	5	.121374	.124511	.117753	.105446	.107823	.000000	.423092
	6	.167625	.107233	.108018	.101260	.098364	.000000	.417499
	7	.372743	.139017	.077841	.079409	.073437	.000000	.257552
	8	.131202	.363359	.129634	.068457	.070026	.000000	.237322
	9	.122256	.104217	.357550	.122256	.061079	.000000	.232643
	10	.114756	.113186	.114756	.328873	.036324	.078431	.213674
	(1, 11)	.128147	.131284	.129716	.131284	.051284	.313725	.114560

When $D = 10$ or (1, 11), the dealer immediately looks at his hole card. If he holds a natural, he announces it at once and proceeds with the settlement. Consequently, if the dealer holds a natural, the player has no opportunity to draw, double down, or split pairs. As a result the player examines these decisions secure in the knowledge that the dealer does not have a natural. Therefore, the important dealer's probabilities for $D = 10$ and (1, 11) are $P(T=t/T \neq$ natural 21), the conditional probabilities for various outcomes t given that the dealer does not obtain a natural. They are given below.

		t					natural 21	>21
		17	18	19	20	21		
D	10	.124522	.122819	.124522	.356862	.039415	.000000	.231859
	(1, 11)	.186728	.191299	.189015	.191299	.074728	.000000	.166930

The entries in the tables of approximate values for $P(T=t)$ have been carried to six places to facilitate a comparison with the exact probabilities in the two cases mentioned and to improve certain numerical checks. Actually, the digits in the fourth and higher places are relatively meaningless and have been dropped in the final results for the mathematical expectation.

IV. METHODS OF ANALYSIS FOR SPECIAL SITUATIONS

A. *Soft Hands*

The method of analysis for all special situations requires tables of $P(H=h/H_p=h_p)$, the conditional probability the player obtains a final total of $h\{h \geqq M(D)\}$ given that he has a partial total of $h_p\{h_p(\text{unique}) < M(D), h_p(\text{soft}) < M^*(D)\}$ and draws or stands following $M(D)$ and $M^*(D)$. The tables were worked out with the same assumptions of equiprobability and sampling with replacement as in the corresponding tables of conditional probabilities for the dealer. Separate tables were required for each of the following four cases: $D = 2, 3$ where $M(D) = 13$ and $M^*(D) = 18$; $D = 4, 5, 6$ where $M(D) = 12$ and $M^*(D) = 18$; $D = 7, 8, (1, 11)$ where $M(D) = 17$ and $M^*(D) = 19$; and $D = 9, 10$ where $M(D) = 17$ and $M^*(D) = 19$.

In the case of soft hands, the decision equation has been used to show that

$M^*(D)>17$, and subsequent analysis need only compare the mathematical expectation of player 1 with $M^*=x$ versus that of player 2 with $M^*=x+1$ for $x\geqq 18$. The two players will use the same strategy except where their total is soft x. In this situation player 1 stands and player 2 draws one card and continues to draw so long as his total is less than $M(D)$. A comparison of the mathematical expectations of player 1 and 2 for soft hands is equivalent to comparing $E_{s,x}$, the expectation of a player standing on a total of x, with $E_{d^*,x}$, the expectation of a player with a soft total of x who draws one card and then draws or stands according to $M(D)$. $E_{s,x}$ has already been calculated. Adopting the convention that $E_{s,k}=-1$ for $k>21$, $E_{d^*,x}$ is given by

$$E_{d^*,x} = \sum_{j\geqq M} P(J=j)E_{s,j} + \sum_{j<M} P(J=j) \sum_{h\geqq M} P(H=h/H_p=j)E_{s,h}.$$

While x does not appear explicitly in the right hand side above, its value affects $P(J=j)$. Since $E_{s,19}>E_{d^*,19}$ for all D, it was not necessary to evaluate $E_{s,x}$ and $E_{d^*,x}$ for $x\geqq 20$.

B. *Doubling Down*

The player who elects to double down on a total of x turns up his hole cards, doubles his bet, and receives one and only one card face down. His expectation then is 2 $E_{d,x}$. To determine whether he should double down the player must compare this expectation with $E_{M,M^*;x}$, the mathematical expectation of the player with a total of x who follows the drawing strategy given by M and M^*. Thus the player will double down if and only if $2E_{d,x}-E_{M,M^*;x}>0$. But $E_{M,M^*;x}\geqq E_{d,x}$, therefore

$$2E_{d,x} - E_{M,M^*;x} \geqq 2E_{d,x} - E_{d,x} = E_{d,x}.$$

This result and the equation for $E_{d,x}$ can be used to show immediately that doubling down is poor strategy where $x(\text{unique})>11$ or <8 for all D and $x(\text{soft})<17$ for $D>6$ and $D=(1, 11)$. Many combinations of D and x need to be examined individually, however, using the following formula for $E_{M,M^*;x}$.

$$E_{M,M^*;x} = \begin{cases} E_{s,x} & x(\text{unique}) \geqq M \text{ or } x(\text{soft}) \geqq M^* \\ \sum_{h\geqq M} P(H=h/H_p=x)E_{s,h} & \text{otherwise.} \end{cases}$$

Define $X=X(D)$ as the set of values of x for which the player should double down when the dealer's up card is D.

C. *Splitting Pairs*

In the case of splitting a pair of y's a separate analysis is required for every combination of y and D. The results for doubling down must be used inasmuch as a player may split a pair and subsequently double down. The player will split his y's if and only if $E_{\text{split},y}>E_{\text{no-split},y}$ where

$$E_{\text{no-split},y} = \begin{cases} 2E_{d,2y} & 2y \in X \\ E_{M,M^*;2y} & 2y \notin X \end{cases}$$

$$\tfrac{1}{2}E_{\text{split},y} = \sum_{j\in X} P(J=j)2E_{d,j} + \sum_{j\notin X} P(J=j)E_{M,M^*;j}.$$

In the special case where $y=(1, 11)$, the player splitting is allowed to draw only one more card and cannot double down. Thus $\frac{1}{2}E_{\text{split},(1,11)}=\sum_{all\,j}P(J=j)E_{s,j}$. Let $Y=Y(D)$ denote the values of y for which a pair of y's should be split when the dealer's up card is D.

V. THE PLAYER'S MATHEMATICAL EXPECTATION

The mathematical expectation of the player is given by

$$E(W) = 1/13 \sum_{D\neq 10} E(W_D) + 4/13E(W_{10}),$$

where W_D is the amount won by the player on a single hand when the dealer's up card is D. When $D=10$ and $(1, 11)$ one must calculate $E(W_D)$ under two conditions: (1) given the dealer does not obtain a natural, and (2) given that he does. These conditional expectations are then multiplied by the probabilities for events (1) and (2) to obtain $E(W_D)$.

The first step in obtaining $E(W_D)$ is to calculate the probabilities for the various hands formed by the player's two hole cards. It is assumed that the hole cards are drawn from a deck which is complete except for one D-counting card. In the case where $D=10$ or $(1, 11)$ and the dealer has a natural

$$E(W_D) = -1[1 - P(\text{the hole cards form a natural})].$$

In all other cases $E(W_D)$ is the sum of the following four terms:

(1) $1\frac{1}{2}\ P$ (the hole cards form a natural),

(2) $\sum_{y\in Y} P$ (the hole cards are a pair of y's) $E_{\text{split},y}$,

(3) $\sum_{j\in X} P$ (the hole cards total j) $2E_{d,j}$,

(4) $\sum_{j\notin X} P$ (the hole cards total j) $E_{M,M^*;j}$.

In the last two sums it is understood that the hole cards do not form a natural or a pair of y's with $y\in Y$.

VI. DESCRIPTION OF THE OPTIMUM STRATEGY

The player's basic problems of strategy are (a) when to draw and stand, (b) when to double down, and (c) when to split pairs. These problems will be treated in order. A fourth subsection on the player's mathematical expectation is also included.

(a) *Drawing Strategy.* Ordinarily, the player's hand has one unique total not exceeding 21. However, sometimes the player will hold a hand with two possible totals not exceeding 21, e.g., an ace and a five is either 6 or 16. In gambling terminology this ambiguous type of hand is known as "soft" and requires a separate strategy.

Some notation will facilitate the description of the optimum strategy for drawing. Let D be the numerical value of the dealer's up card. $D=2, 3, \cdots, 10, (1, 11)$. Let $M(D)$ be an integer such that if the dealer's up card is D and the player's total is unique and less than $M(D)$, the player should draw; while if

the player's total is unique and greater or equal to $M(D)$, the player should stand. The ten integers $M(D)$ are known as the minimum standing numbers for unique hands. Let us define $M^*(D)$ in the same way for soft hands with the understanding that "player's total" means the larger of the two possible totals. The optimum strategy may now be described as follows:

$$M(D) = \begin{cases} 13 & D = 2, 3 \\ 12 & D = 4, 5, 6 \\ 17 & D \geqq 7, D = (1, 11) \end{cases} \qquad M^*(D) = \begin{cases} 18 & D \leqq 8, D = (1, 11) \\ 19 & D = 9, 10 \end{cases}$$

The most surprising aspect of the optimum strategy for unique hands is the low values of $M(D)$ for $D \leqq 6$. Few experienced players recommend standing on a total of 13 under any circumstances, and standing on 12 would be completely out of the question. Culbertson, Morehead, and Mott-Smith, [1] for example, propose a strategy of $M(D) = 14$ for $D \leqq 6$ and $M(D) = 16$ for $D \geqq 7$ and $D = (1, 11)$. Also unexpected is the great "discontinuity" in $M(D)$ as D goes from 6 to 7. The optimum strategy for soft hands is not particularly surprising except perhaps for $D = 9, 10$. Most experts, including Culbertson et al., suggest $M^*(D) = 18$ for all D.

(b) *Doubling Down.* The optimum strategy is given in the following table.

Player's *Unique* Two Card Total	≧12	11	10	9	≦8
Values of D Where the Player Should Double Down	none	$2 \leqq D \leqq 10$	$2 \leqq D \leqq 9$	$2 \leqq D \leqq 6$	none
Player's *Soft* Two Card Total	≧19	18	17	13–16	12
Values of D Where the Player Should Double Down	none	$D = 4, 5, 6$	$D = 3, 4, 5, 6$	$D = 5, 6$	$D = 5$

Note that two card soft 12 always consists of two aces. While the above table shows it is good strategy to double down on soft 12 when the dealer shows a five, more detailed analysis shows that it is even better strategy to split the aces. Similarly, while it is good strategy to stand rather than draw on two card soft 18 when $D = 4, 5, 6$, it is even better strategy to double down.

The fact that the player should double down so frequently may surprise many people. Doubling down is not common in the casinos, and, in particular, the idea of doubling down on soft hands probably does not occur to most players. Doubling down is not recommended by Culbertson et al. nor by any other writers on Blackjack encountered by the authors [2, 3, 4]. One may even go so far as to say that doubling down is the most neglected, under-rated aspect of Blackjack strategy.

(c) *Splitting Pairs.* The optimum strategy is given in the following table.

Type of Pair	ace's, 8's	9's	7's	6's, 3's, 2's	4's	10's, 5's, face cards
Values of D Where all values The Pair is Split		$2 \leqq D \leqq 6$ $D = 8, 9$	$2 \leqq D \leqq 8$	$2 \leqq D \leqq 7$	$D = 5$	no values

The fact that the optimum strategy calls for splitting aces will evoke no surprise as this is common strategy in the casinos, and is the only split always

recommended by the experts. (Culbertson et al. believe that aces are the only pair which should be split.) In other respects, however, the optimum strategy is quite surprising. Splitting eights, for example, is seldom seen in the casinos, while splitting tens and face cards is not uncommon. Furthermore, the detailed strategy for splitting nines, sevens, sixes, fours, threes and twos probably defies the intuition of even the most experienced players.

(d) *The Player's Mathematical Expectation.* Let W, a random variable, be the amount won by the player on a single hand under the following conditions: the player bets one unit of capital, the game of blackjack is defined by the rules given in Section I, and the player utilizes the optimum strategy described in (a), (b), (c) above. *W has mathematical expectation $-.006$ and variance 1.1.*

While the player's overall expectation is $-.006$, $E(W_D)$, the player's conditional expectation given that the dealer's up card is D, shows considerable variability and is positive for seven out of the ten values of D.

D	2	3	4	5	6	7	8	9	10	(1, 11)
$E(W_D)$	.090	.123	.167	.218	.230	.148	.056	$-.043$	$-.176$	$-.363$

The player's disadvantage in blackjack stems entirely from the rule that if both the player and dealer bust, the dealer wins. The player's disadvantage is smaller than in other popular house games such as craps and roulette where his mathematical expectation is, at best, $-.014$. Furthermore, the optimum strategy is considerably superior to many common strategies. The player, for example, who follows the strategy recommended by Culbertson et al. has an expectation of $-.036$. The player who mimics the dealer, drawing to 16 or less, standing on 17 or more, never doubling down or splitting pairs, has an expectation of $-.056$.

The optimum strategy was developed under the assumption that the player does not have the time or inclination to utilize the information available in the hands of the players preceding him in the draw. This information is nonexistent when the player sits on the dealer's left and is greatest when the player is on the dealer's right. There are tremendous difficulties, however, in using this information except in an intuitive, non-scientific manner.

Practical considerations required that the optimum strategy be developed under the above major assumption and several less important assumptions discussed in Sections II-V. Because of the need for these simplifying assumptions, the strategy presented in this section could be more precisely described as a "practical" optimum strategy.

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TWO SEQUENTIAL TESTS AGAINST TREND*

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Two procedures are suggested for testing the hypothesis of randomness against alternatives of a linear trend and a cyclical trend, respectively. Both procedures reduce to tests of the hypothesis that the parameter of a binomial distribution is p_0 against the alternative that it is p_1 and can, therefore, be easily sequentialized. The expected number of observations required by the sequential test against linear trend is smaller than the corresponding number of observations of other tests in current use.

1. INTRODUCTION

THE great majority of statistical procedures in current use start with the assumption that we are given a random sample from some population, i.e., from the assumption that the joint distribution $F(x_1, x_2, \cdots, x_n)$ of the given observations $x_1, x_2, \cdots, x_n$ is the product of n identical distribution functions,

$$F(x_1, x_2, \cdots, x_n) = F(x_1)F(x_2) \cdots F(x_n). \tag{1}$$

Despite the importance of this assumption, it is rarely brought to a test, possibly because it is often felt that too large a number of observations is required to carry out a successful investigation. For this reason, sequential tests of randomness may have more appeal.

An alternative to randomness may take a great many different forms. It is obvious that no single test of randomness can be satisfactory with respect to all possible alternatives. Thus, it is necessary to select certain particular alternatives to randomness and devise tests with these alternatives in mind.

In this paper we shall consider two alternatives. a) Linear trend:

$$F(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n} F(x_i + i\theta), \tag{2}$$

$F(x)$ being an arbitrary distribution function and θ, a constant. We have an upward trend if $\theta<0$, a downward trend if $\theta>0$. b) Cyclical trend:

$$F(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n} F(x_i + \theta_i), \tag{3}$$

where the θ_i follow some irregular cyclical movement.

Throughout the paper it will be assumed that the distribution functions involved are continuous.

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2. THE S_j-TEST AGAINST LINEAR TREND

2.1. *Description of Test.* Let j be a positive integer. Given observations $x_1, x_2, \cdots$, define

$$y_m = \begin{matrix} 1 \\ 0 \end{matrix} \text{ if } x_m \begin{matrix} > \\ < \end{matrix} x_{j+m}, \qquad m = 1, 2, \cdots, j.$$

Let p stand for the probability that y_m takes the value 1,

$$p = P(y_m = 1) = P(x_m > x_{j+m}) = P(x_1 > x_{j+1}).$$

Under the null hypothesis H_0 given by (1), $p = p_0 = 1/2$. Under the alternative hypothesis H_1 of the linear trend (2)

$$p = p_1 = \int_{-\infty}^{\infty} F(x + j\theta)dF(x) = 1/2 + \epsilon_j, \text{ say.} \tag{4}$$

Thus, our test of randomness can be reduced to a test of the hypothesis H_0': $p = p_0 = 1/2$, against the alternative H_1': $p = p_1$, where p is the parameter of the binominal distribution associated with the y's. The symbol S_j will be used to denote the test of H_0 against the alternative H_1 which consists in testing H_0' against H_1' by means of the appropriate Wald sequential probability ratio test, truncated, if necessary, when m reaches the value j.

The Wald sequential probability ratio test is completely determined by the values of p_0 and p_1 and the corresponding probabilities α and β of errors of the first and second kinds, respectively. In our case, we always have $p_0 = 1/2$, while p_1 is given by (4). For an upward trend, $p_1 < 1/2$; for a downward trend, $p_1 > 1/2$. For simplicity, we shall assume from now on that we are dealing with a downward trend. However, the results are equally valid for upward trends.[1]

The usual formulas for the intercepts and slope of the two parallel lines determining the Wald probability ratio test can now be written as[2]

$$h_0 = \frac{\log \dfrac{\beta}{1-\alpha}}{\log \dfrac{1+2\epsilon}{1-2\epsilon}}, \quad h_1 = \frac{\log \dfrac{1-\beta}{\alpha}}{\log \dfrac{1+2\epsilon}{1-2\epsilon}}, \quad s = \frac{-\log(1-2\epsilon)}{\log \dfrac{1+2\epsilon}{1-2\epsilon}} \tag{5}$$

where $\epsilon = \epsilon_j$ is defined by (4). The corresponding expected numbers of y-observations under H_0' and H_1' are given by

$$\bar{n}_0 = \frac{2C_0}{\log(1-4\epsilon^2)}, \quad \bar{n}_1 = \frac{2C_1}{\log(1-4\epsilon^2) + 2\epsilon \log \dfrac{1+2\epsilon}{1-2\epsilon}} \tag{6}$$

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[1] The simplest way to deal with an upward trend is to set $z_m = 1 - y_m$ and perform the S_j-test for a downward trend on the basis of z-observations instead of y-observations.

[2] See, e.g., [6, Chapter 5].

where

$$C_0 = (1-\alpha)\log\frac{\beta}{1-\alpha} + \alpha\log\frac{1-\beta}{\alpha},$$

$$C_1 = \beta\log\frac{\beta}{1-\alpha} + (1-\beta)\log\frac{1-\beta}{\alpha}.$$

Table 1 gives values of $h=-h_0=h_1$, s, $\tilde{n}_0$, and $\tilde{n}_1$ as computed from (5) and (6) for $\epsilon=.01(.01).49$ and $\alpha=\beta=.05$.[3]

Having described the S_j-test, the question arises how the value of j to be used in a particular case should be determined. This question will be investigated in the next section. At this point, we only want to note that the possible truncation of the S_j-test at $m=j$ will not appreciably change the performance of the test, provided j is 2 or 3 times as large as $\tilde{n}_t$, $t=0, 1$.

2.2 *Determination of* j. A reasonable criterion for selecting j seems to be to minimize the expected number of x-observations, $\tilde{n}_t(S_j)$, say. Obviously,

$$n_t(S_j) < j + \tilde{n}_t, \qquad t = 0, 1, \tag{7}$$

where $\tilde{n}_t$ is given by (6). We have to use the $<$ sign because of truncation. Actually, if j is considerably larger than $\tilde{n}_t$, the two sides of (7) differ very little.

It is then suggested that j be determined in such a way as approximately to minimize the right side of (7). Such optimum values of j clearly exist, since $\tilde{n}_t$ decreases as j increases. If the optimum values of j differ for $t=0$ and $t=1$, a compromise value has to be chosen.

The attempt to minimize the right side of (7) with respect to j raises the following problem. According to (6) and (4), $\tilde{n}_t$ depends on $\epsilon=\epsilon_j$, which in turn depends not only on the displacement $j\theta$, but also on the underlying distribution $F(x)$. Since $F(x)$ is in general unknown, the question arises how to determine ϵ.

In order to investigate the dependence of ϵ on $F(x)$, set $j\theta/\sigma=\rho_j=\rho$, where σ is the standard deviation of $F(x)$, and compute ϵ when $F(x)$ is the uniform and normal distribution, respectively. For the uniform distribution,

$$\epsilon = \begin{cases} \dfrac{\rho}{2\sqrt{3}}\left(1-\dfrac{\rho}{4\sqrt{3}}\right) & 0 \leqq \rho \leqq 2\sqrt{3}, \\ \dfrac{1}{2} & 2\sqrt{3} \leqq \rho < \infty, \end{cases} = U(\rho), \quad \text{say},$$

while for the normal distribution,

$$\epsilon = \Phi\left(\frac{\rho}{\sqrt{2}}\right) - \frac{1}{2} = N(\rho), \quad \text{say},$$

where $\Phi(t)$ is the standard normal *cdf*.

[3] The author is obliged to Mr. D. R. Morrison for computing Table 1, as well as Table 2.

TABLE 1

ϵ	h	s	$\overline{n}_0$	$\overline{n}_1$
0.01	73.5947	0.5050	13364.65	13131.48
0.02	36.7873	0.5100	3303.26	3318.49
0.03	24.5074	0.5150	1469.66	1471.23
0.04	18.3637	0.5200	825.25	827.48
0.05	14.6731	0.5250	527.37	529.10
0.06	12.2093	0.5301	365.49	367.07
0.07	10.4468	0.5351	267.71	269.56
0.08	9.1224	0.5402	204.35	206.17
0.09	8.0900	0.5453	160.91	162.70
0.10	7.2619	0.5503	129.83	131.61
0.11	6.5825	0.5555	106.83	108.61
0.12	6.0146	0.5606	89.34	91.11
0.13	5.5202	0.5658	75.55	77.33
0.14	5.1175	0.5710	64.91	66.70
0.15	4.7565	0.5762	56.20	57.99
0.16	4.4391	0.5814	49.06	50.85
0.17	4.1577	0.5867	43.14	44.94
0.18	3.9063	0.5921	38.18	39.98
0.19	3.6800	0.5975	33.98	35.79
0.20	3.4752	0.6029	30.40	32.21
0.21	3.2884	0.6084	27.31	29.12
0.22	3.1176	0.6139	24.63	26.45
0.23	2.9604	0.6195	22.29	24.12
0.24	2.8150	0.6252	20.24	22.06
0.25	2.6807	0.6309	18.43	20.26
0.26	2.5575	0.6367	16.83	18.67
0.27	2.4368	0.6427	15.37	17.22
0.28	2.3264	0.6486	14.09	15.94
0.29	2.2223	0.6548	12.92	14.79
0.30	2.1240	0.6610	11.88	13.75
0.31	2.0306	0.6673	10.92	12.81
0.32	1.9418	0.6738	10.06	11.95
0.33	1.8569	0.6804	9.27	11.17
0.34	1.7757	0.6871	8.54	10.45
0.35	1.6975	0.6941	7.87	9.80
0.36	1.6220	0.7012	7.25	9.20
0.37	1.5489	0.7086	6.68	8.64
0.38	1.4778	0.7163	6.15	8.13
0.39	1.4083	0.7242	5.70	7.70
0.40	1.3401	0.7325	5.19	7.20
0.41	1.2726	0.7412	4.75	6.78
0.42	1.2056	0.7503	4.33	6.40
0.43	1.1383	0.7601	3.94	6.03
0.44	1.0701	0.7706	3.56	5.68
0.45	1.0000	0.7820	3.19	5.36
0.46	0.9265	0.7947	2.83	5.05
0.47	0.8471	0.8094	2.46	4.75
0.48	0.7566	0.8271	2.08	4.46
0.49	0.6408	0.8513	1.64	4.16

Constants for Wald Sequential Probability Ratio Test for Binomial Distribution: $p_0 = \frac{1}{2}$ against $p_1 = \frac{1}{2} + \epsilon$, $\alpha = \beta = .05$.

($\pm h$ = intercepts, s = slope of parallel lines; $\overline{n}_0$ and $\overline{n}_1$ = expected number of observations when $p = p_0$ and $p = p_1$, respectively.)

Computations show that the absolute difference $|U(\rho)-N(\rho)|$ is always smaller than .02, and for $\rho<1$ and $\rho>2$ is considerably smaller. Since the uniform and normal distributions are quite different in character, this would seem to indicate that, in general, the values of ϵ_j do not depend too much on the form of $F(x)$ as long as the displacement parameter is measured in units of the standard deviation.

Thus we might get the ϵ_j-values necessary for minimizing the right side of (7) from the normal distribution, i.e., by setting $\epsilon_j=N(\rho)$, $\rho=j\theta/\sigma$. Values of $N(\rho)$ are given in Table 2 for $\rho=.01(.01)1.00(.02)2.00(.04)3.00$.

In order to be able to apply the S_j-test, we have to specify a value θ_1 of the parameter θ determining the downward trend, such that if θ is actually equal to θ_1, we should like our test to reject the hypothesis of randomness with a given probability $1-\beta$. As an example, let us take $\theta_1=\sigma/25$ where σ is the standard deviation of the underlying distribution. Let us also choose $\alpha=\beta=.05$, where, of course, α is the probability of committing an error of the first kind. The value of j to be used in the S_j-test can now easily be found with the help of Tables 1 and 2.

For this purpose, we find ϵ_j for various values of j by entering Table 2 with $\rho=j\theta_1/\sigma=.04_j$. Entering Table 1 with these values of ϵ_j, we find the corresponding values of $\bar{n}_0$ and $\bar{n}_1$. In this way, we obtain the following table:

j	ϵ_j	$\bar{n}_0$	$j+\bar{n}_0$	$\bar{n}_1$	$j+\bar{n}_1$
24	.251	18.3	42.3	20.1	44.1
26	.269	15.5	41.5	17.4	43.4
28	.286	13.4	41.4	15.2	43.2
30	.302	11.7	41.7	13.6	43.6
32	.317	10.3	42.3	12.2	44.2

It is seen that $j+\bar{n}_0$ and $j+\bar{n}_1$ are approximately minimized for several consecutive values of j. Since the probability that truncation becomes necessary decreases rapidly as $j/\bar{n}_i$ increases, it will be best to select a rather high value of j among those which approximately minimize (7). Thus, in this case, $j=30$ would seem to be a good choice.

Similarly, the following values of j are suitable choices for the corresponding values of θ_1/σ:

θ_1/σ	j	$j+\bar{n}_0$	$j+\bar{n}_1$
.10	17	23	25
.06	23	32	34
.04	30	42	44
.03	35	50	52
.02	45	66	68
.01	70	104	10[illegible]

The table also gives the expected number of x-observations required by the corresponding S_j-test under H_0 and H_1 for $\alpha=\beta=.05$. Perhaps, it should be added that any S_j-test requires always at least $j+1$ observations.

2.3. *Example.* Hald [3, p. 350] describes an experiment designed to measure the effectiveness of a certain mixer. The mixing process is called effective if

TABLE 2

ρ	$N(\rho)$	ρ	$N(\rho)$	ρ	$N(\rho)$	ρ	$N(\rho)$
0.01	0.00283	0.46	0.12752	0.91	0.24005	1.72	0.38801
0.02	0.00562	0.47	0.13017	0.92	0.24232	1.74	0.39065
0.03	0.00846	0.48	0.13285	0.93	0.24460	1.76	0.39343
0.04	0.01129	0.49	0.13502	0.94	0.24688	1.78	0.39598
0.05	0.01412	0.50	0.13818	0.95	0.24914	1.80	0.39849
0.06	0.01691	0.51	0.14080	0.96	0.25136	1.82	0.40095
0.07	0.01974	0.52	0.14345	0.97	0.25361	1.84	0.40347
0.08	0.02257	0.53	0.14610	0.98	0.25585	1.86	0.40575
0.09	0.02536	0.54	0.14870	0.99	0.25804	1.88	4.40808
0.10	0.02818	0.55	0.15132	1.00	0.26022	1.90	0.41053
0.11	0.03101	0.56	0.15395	1.02	0.26461	1.92	0.41277
0.12	0.03383	0.57	0.15656	1.04	0.26895	1.94	0.41497
0.13	0.03661	0.58	0.15913	1.06	0.27322	1.96	0.41713
0.14	0.03943	0.59	0.16173	1.08	0.27748	1.98	0.41924
0.15	0.04225	0.60	0.16429	1.10	0.28166	2.00	0.42132
0.16	0.04502	0.61	0.16687	1.12	0.28582	2.04	0.42563
0.17	0.04784	0.62	0.16945	1.14	0.28991	2.08	0.42935
0.18	0.05065	0.63	0.17202	1.16	0.29395	2.12	0.43306
0.19	0.05346	0.64	0.17455	1.18	0.29797	2.16	0.43662
0.20	0.05622	0.65	0.17710	1.20	0.30192	2.20	0.44015
0.21	0.05903	0.66	0.17964	1.22	0.30585	2.24	0.44340
0.22	0.06183	0.67	0.18218	1.24	0.30970	2.28	0.44652
0.23	0.06458	0.68	0.18467	1.26	0.31354	2.32	0.44950
0.24	0.06738	0.69	0.18719	1.28	0.31727	2.36	0.45244
0.25	0.07017	0.70	0.18970	1.30	0.32100	2.40	0.45515
0.26	0.07292	0.71	0.19217	1.32	0.32469	2.44	0.45774
0.27	0.07570	0.72	0.19466	1.34	0.32831	2.48	0.46028
0.28	0.07848	0.73	0.19714	1.36	0.33190	2.52	0.46263
0.29	0.08125	0.74	0.19962	1.38	0.33542	2.56	0.46485
0.30	0.08399	0.75	0.20205	1.40	0.33891	2.60	0.46697
0.31	0.08675	0.76	0.20450	1.42	0.34231	2.64	0.46905
0.32	0.08952	0.77	0.20692	1.44	0.34566	2.68	0.47095
0.33	0.09224	0.78	0.20935	1.46	0.34896	2.72	0.47276
0.34	0.09499	0.79	0.21178	1.48	0.35245	2.76	0.47453
0.35	0.09774	0.80	0.21420	1.50	0.35566	2.80	0.47615
0.36	0.10048	0.81	0.21661	1.52	0.35881	2.84	0.47768
0.37	0.10319	0.82	0.21898	1.54	0.36192	2.88	0.47912
0.38	0.10592	0.83	0.22136	1.56	0.36499	2.92	0.48054
0.39	0.10865	0.84	0.22374	1.58	0.36800	2.96	0.48183
0.40	0.11134	0.85	0.22608	1.60	0.37097	3.00	0.48304
0.41	0.11405	0.86	0.22844	1.62	0.37410		
0.42	0.11677	0.87	0.23079	1.64	0.37688		
0.43	0.11948	0.88	0.23313	1.66	0.37980		
0.44	0.12214	0.89	0.23542	1.68	0.38259		
0.45	0.12483	0.90	0.23774	1.70	0.38532		

$$N(\rho) = \Phi(\rho/\sqrt{2}) - 1/2, \qquad \Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-x^2/2}\, dx.$$

consecutive measurements of the concentration of a given substance exhibit the properties of a random sample. It is called ineffective, if they exhibit the properties of a downward trend. Thus we have the case of testing for randomness against the alternative of a downward trend.

According to our previous discussion, the S_j-test to be used depends on three quantities: θ_1, α, and β. Let us again take $\theta_1=\sigma/25$, $\alpha=\beta=.05$. Then, as we have just seen, the S_{30}-test is appropriate. The choice of $\theta_1=\sigma/25$ may seem rather arbitrary, and it certainly is as far as this example is concerned. However, in practice, it certainly is reasonable to assume that the investigator has some idea of the amount of change in location which he would like the test to detect with high probability.

Hald gives 58 observations. The corresponding y-values for $j=30$ become:

1 1 1 1 1 0 0 1 — 0 1 1 1 1 1 1 1 1 1 1 1 — 1 1 1 1 1 1 0.

A dash indicates that the corresponding two x-observations are equal, presumably due to limitations of the measuring technique. In our analysis we shall omit such observations. Of course, the number of x-observations required by the S_j-test is increased in this way by 1 for each tie.

Entering Table 1 with $\epsilon=\epsilon_{30}=.302$, we find $h=2.1$ and $s=.66$ leading to the sequential test represented by Fig. 1. The hypothesis that the mixing process is effective is rejected after 16 y-observations. In view of the ties, this corresponds to 47 x-observations. Thus, if the S_{30}-test is used, the last 11 observations are not needed.

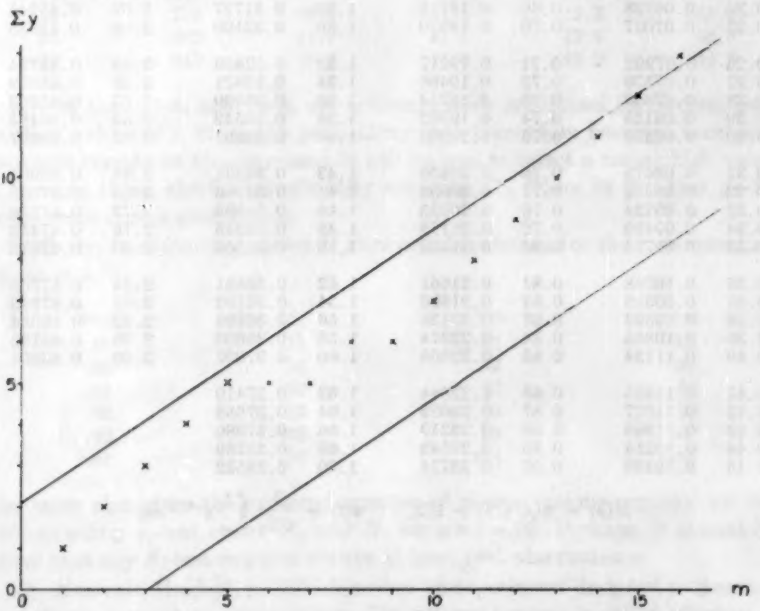

FIG. 1. S_{30}-test on data [3, p. 351], $\alpha=\beta=.05$, $\theta_1=\sigma/25$.

Had the investigator felt that the test should detect a trend corresponding to $\theta=\sigma/50$ with probability .95 when using a significance level of .05, the S_{45}-test would have been appropriate. In that case, the y-observations become:

$$1\ 1\ 1\ 1\ 1\ 1 - 1\ 1\ 0\ 1\ 1\ 1.$$

The values of h and s corresponding to $\epsilon_{45}=.238$ are 2.8 and .62, respectively, leading to the rejection of the null hypothesis after 8 y-observations or 54 x-observations.

Hald, in analyzing the data, uses runs above and below the median. The total number of runs is not significant, while the length of the longest runs is highly significant. Hald concludes that the mixing has not been quite satisfactory.

2.4. *Comparison with Other Tests.* A great many distribution-free tests have been devised to test for randomness against the hypothesis of a downward trend. Stuart [5], Foster and Stuart [2], Cox and Stuart [1] have carried out comparisons among many of these tests. From their investigations it would seem that the two tests based on Kendall's and Spearman's rank correlation coefficients, respectively, are best from the point of view of power and asymptotic efficiency. Since the relative asymptotic efficiency of these two tests with respect to each other is 1, it is sufficient to compare the S_j-test with either test. It can be shown that if θ_1/σ is sufficiently small, there exist values j such that the expected number of observations required by the corresponding S_j-test is smaller than the corresponding number of observations required by the test based on Kendall's rank correlation coefficient. Thus, if j is selected as suggested in a previous section, the corresponding S_j-test will require, on the average, fewer observations than the tests investigated by Stuart *et al.*

It may be objected that the S_j-test is a test specifically designed to detect a linear trend, while the rank correlation tests are designed for any kind of downward trend. While no computations have been carried out, the author believes that the S_j-test remains reasonably effective against general downward trends, if θ_1 is interpreted as an average shift in the location parameter.

Besides requiring fewer observations than other tests, the S_j-test has other advantages. Once the value of j is determined, the application of the test is extremely simple, requiring no lengthy computations and/or orderings of observations as is usually the case with rank order tests. In addition, approximate operating characteristics and average sample numbers as functions of θ/σ can be computed from the corresponding formulas for the sequential test concerning the parameter p of a binomial distribution. This is in sharp contrast to other tests of randomness for which often not much more than the distribution under the hypothesis to be tested is known.

In conclusion, the author wants to disagree with the statement made by Cox and Stuart [1] that sequential tests, like the S_j-test, are likely to be of practical value only under exceptional circumstances. The example from Hald does not seem to be very exceptional and is certainly ready-made for the S_j-test.

3. A SEQUENTIAL RUN-TEST AGAINST CYCLIC ALTERNATIVES

3.1. *Description of Test.* Let the alternative to randomness be a cyclical change of type (3). The following test is suggested. Divide observations into groups of length 3 in the order in which they are taken. Associate with each group a chance variable y which takes the value 1 or 0 depending on whether the corresponding three x-observations form a monotonic sequence or not.[4] If we set $P(y=1)=p$, then p is the parameter of a binomial distribution, and the test of randomness reduces to a test of the hypothesis $p=p_0$ against $p=p_1>p_0$, which can be carried out sequentially in the well-known manner.

Since 3 observations can be arranged in 6 different ways, all of which are equally likely under the hypothesis of randomness, and since only 2 of these 6 arrangements are monotonic, $p_0=1/3$. (It is assumed that no two observations are equal.) In order to carry out the test, it is necessary to select a value p_1 such that if actually $p=p_1$ we should like to have probability $1-\beta$ of rejecting the hypothesis of randomness. For reasons which will be given in Section 3.3, $p_1=1/2$ may often be a suitable choice.

For $p_0=1/3$, $p_1=1/2$, the slope of the two parallel lines determining the Wald sequential probability ratio test is given by $s=.41504$. The two intercepts depend also on the quantities α and β. For various choices of α and β, they are found to be:

α	β	h_0	h_1
.01	.001	−9.95	6.64
.01	.01	−6.63	6.63
.01	.05	−4.31	6.57
.05	.001	−9.89	4.32
.05	.01	−6.57	4.31
.05	.05	−4.25	4.25
.05	.10	−3.25	4.17
.10	.01	−6.49	3.31
.10	.05	−4.17	3.25
.10	.10	−3.17	3.17

3.2. *Example.* Hald [3, p. 356] gives an example involving 60 consecutive measurements of the quality of a certain product. It is suspected that periodic fluctuations in the quality of the raw material produce similar fluctuations in the quality of the finished product. Thus we have the problem of testing for randomness against the alternative of cyclical changes. The 60 observations produce the following set of y-observations:

0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0.

From the description of the problem, it seems reasonable to choose a fairly high value of α, coupled with a low value of β. Fig. 2 represents the corresponding test for $\alpha=.10$, $\beta=.01$. It is seen that the hypothesis of randomness is rejected after 18 y-observations.[5]

[4] If a + sign is used to denote the fact that $x_{i+1}-x_i>0$, a − sign, that $x_{i+1}-x_i<0$, $y=1$ if the corresponding sequence of x-observations corresponds to a (+ +) or a (− −) arrangement; $y=0$, if it corresponds to a (+ −) or a (− +) arrangement.

[5] The test based on $\alpha=\beta=.05$ which is customarily used in text book examples would not have resulted in outright rejection of H_0 with the available y-observations. However, truncation after 20 observations would have done so.

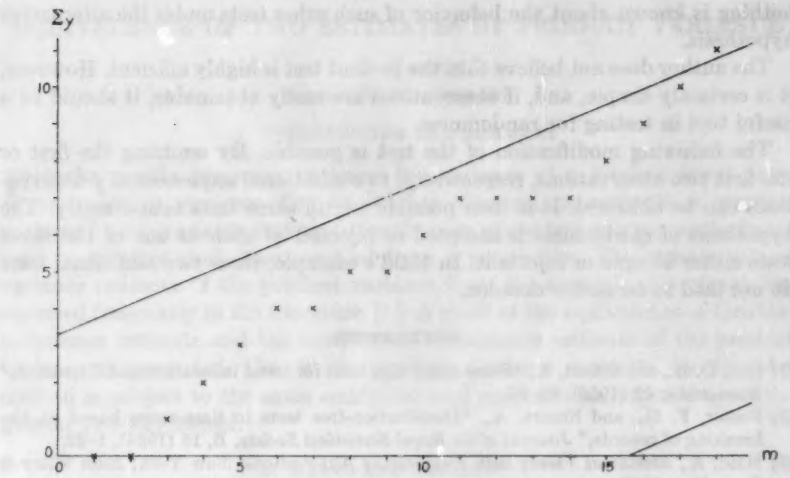

FIG. 2. Sequential run-test on data [3, p. 356], $\alpha = .10$, $\beta = .01$.

Hald uses runs up and down. The total number of runs is not quite significant at the .05 level, while the length of the longest run is. As a consequence, Hald concludes that the finished product reflects the variations in the raw material.

3.3. *General Remarks.* In 3.1, it has been suggested to choose p_1 equal to 1/2. The reasoning in back of this suggestion is as follows. The value of p depends on the function $F(x)$ and the parameters θ_i in (3). We have seen in Section 2.2 that expressions of the kind $P(x_1 > x_2)$ do not depend too much on the underlying distribution $F(x)$ as long as the displacement parameter involved is expressed in units of the standard deviation. Levene [4] has noted that this holds also for expressions of the type $P(x_1 > x_2 > x_3)$. We can get then some idea of the value of p by taking $F(x)$ as the uniform distribution and assuming that x_1, x_2, x_3 follow a linear trend with parameter θ.

Setting $\theta = \rho\sigma$ where σ is the standard deviation of the underlying uniform distribution, it is found that

$$p = \frac{1}{3} + \frac{1}{4}\rho^2 - \frac{\sqrt{3}}{27}\rho^3, \qquad \rho \leqq \sqrt{3}.$$

For $\rho = 0$, we have the hypothesis of randomness, and therefore, $p_0 = 1/3$, as we have seen earlier by means of a different argument. For $\rho = 1$, we find that $p = .52$. Thus, choosing $p_1 = 1/2$ implies that if the change in location from observation to observation is approximately equal to one standard deviation, we have probability $1 - \beta$ of finding out about it.

While it would be possible to write down the operating characteristic and average sample number of the present test as a function of the parameter ρ mentioned above, comparisons of the present test with other tests designed to discover cyclic fluctuations are difficult for the simple reason that practically

nothing is known about the behavior of such other tests under the alternative hypothesis.

The author does not believe that the present test is highly efficient. However, it is certainly simple, and, if observations are easily obtainable, it should be a useful tool in testing for randomness.

The following modification of the test is possible. By omitting the first or the first two observations, respectively, two additional sequences of y-observations can be obtained. It is then possible to run three tests concurrently. The hypothesis of randomness is accepted or rejected as soon as one of the three tests either accepts or rejects it. In Hald's example, these two additional tests do not lead to an earlier decision.

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EQUIVALENCE OF TWO ESTIMATES OF PRODUCT VARIANCE

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1. INTRODUCTION AND SUMMARY

IT IS frequently necessary to know the variance of a product apart from measurement variance. Grubbs [3] has obtained an estimate of product variance by a covariance technique for the case where two or more methods are used to measure each item sampled from a population. The components of variance estimate of the product variance from the analysis of variance has occurred frequently in the literature [1]. A proof of the equivalence of Grubbs' covariance estimate and the components of variance estimate of the product variance is presented. Use of the product variance estimate by Grubbs' method is subject to the same conditions and assumptions imposed upon the general two-way model.

2. MODEL AND NOTATION

Grubbs considers the following model:

$$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$$

where $i=1, 2, \cdots a$ items are measured by $j=1, 2, \cdots b$ methods, with the expected values of α_i, β_j, ϵ_{ij} and all their covariances equal to zero, $E(\alpha_i^2)=\sigma_a^2$ is the product variance and $E(\epsilon_{ij}^2)=\sigma_e^2$ is the error variance.

The following notation will be used:

$$Y_{i\cdot} = \sum_{j=1}^{b} y_{ij}, \qquad Y_{\cdot\cdot} = \sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}, \qquad y_{i\cdot} = \frac{Y_{i\cdot}}{b},$$

$$y_{\cdot\cdot} = \frac{Y_{\cdot\cdot}}{ab}, \text{ and similarly for } Y_{\cdot j} \text{ and } y_{\cdot j}.$$

3. GRUBBS' ESTIMATE

Grubbs [3, p. 261, eq. (30)] gives the following estimate of product variance for "a" items measured by "b" methods:

$$s_a^2 = \frac{s^2_{ba+a_1+\cdots+a_b}}{b^2} - \frac{1}{b^2(b-1)} \sum_{1\leq r<s}^{b} s^2_{t_r - t_s} \tag{1}$$

where Grubbs defines

$$s^2_{ba+a_1+\cdots+a_b} = \frac{b^2}{a-1}\sum_{i=1}^{a}(y_{i\cdot} - y_{\cdot\cdot})^2$$

and

$$s^2_{t_r - t_s} = \frac{1}{a-1}\left[\sum_{i=1}^{a}(y_{ir} - y_{is})^2 - \frac{(Y_{\cdot r} - Y_{\cdot s})^2}{a}\right].$$

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4. COMPONENTS OF VARIANCE ESTIMATE

The components of variance estimate of product variance as given by Anderson and Bancroft [1, p. 319] is:

$$s_a^2 = \frac{1}{b}\sum_{i=1}^{a}\frac{b(y_{i\cdot} - y_{\cdot\cdot})^2}{a-1} - \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{(y_{ij} - y_{i\cdot} - y_{\cdot j} + y_{\cdot\cdot})^2}{b(a-1)(b-1)}. \quad (2)$$

5. PROOF OF THE EQUIVALENCE OF THE TWO ESTIMATES

The first terms of the components of variance estimate (2) and Grubbs' estimate (1) are equivalent. Thus, all that remains to show that the two estimators are equivalent is to show that

$$\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij} - y_{i\cdot} - y_{\cdot j} + y_{\cdot\cdot})^2 = \frac{1}{b}\sum_{1\le r<s}^{b}\left[\sum_{i=1}^{a}(y_{ir} - y_{is})^2 - \frac{(Y_{\cdot r} - Y_{\cdot s})^2}{a}\right]. \quad (3)$$

Consider the RHS of (3);

$$\sum_{1\le r<s}^{b}\sum_{i=1}^{a}(y_{ir} - y_{is})^2 = \frac{1}{2}\sum_{\substack{r,s\\ r\neq s}}^{b}\sum_{i=1}^{a}(y_{ir}^2 + y_{is}^2 - 2y_{ir}y_{is})$$

$$= (b-1)\sum_{r=1}^{b}\sum_{i=1}^{a}y_{ir}^2 - \sum_{r=1}^{b}\sum_{i=1}^{a}y_{ir}(Y_{i\cdot} - y_{ir})$$

$$= b\sum_{r=1}^{b}\sum_{i=1}^{a}y_{ir}^2 - \sum_{i=1}^{a}Y_{i\cdot}^2.$$

and

$$\sum_{1\le r<s}^{b}\frac{(Y_{\cdot r} - Y_{\cdot s})^2}{a} = \frac{1}{2}\sum_{\substack{r,s\\ r\neq s}}^{b}\frac{(Y_{\cdot r} - Y_{\cdot s})^2}{a}$$

$$= \frac{1}{2}\sum_{\substack{r,s\\ r\neq s}}^{b}\frac{(Y_{\cdot r}^2 + Y_{\cdot s}^2 - 2Y_{\cdot r}Y_{\cdot s})}{a}$$

$$= \frac{1}{a}\left[(b-1)\sum_{r=1}^{b}Y_{\cdot r}^2 - \sum_{r=1}^{b}Y_{\cdot r}(Y_{\cdot\cdot} - Y_{\cdot r})\right]$$

$$= \frac{b\sum_{r=1}^{b}Y_{\cdot r}^2 - Y_{\cdot\cdot}^2}{a}.$$

Hence, the RHS of (3) is

$$\sum_{r=1}^{b}\sum_{i=1}^{a}y_{ir}^2 - \sum_{i=1}^{a}\frac{Y_{i\cdot}^2}{b} - \sum_{r=1}^{b}\frac{Y_{\cdot r}^2}{a} + \frac{Y_{\cdot\cdot}^2}{ab} \quad (4)$$

which is equal to the LHS of (3). The equivalence of the two forms for writing the Interaction Sum of Squares on the LHS of (3) and in (4) is given by Bennett and Franklin [2, p. 371]. Thus, (1) and (2) are equivalent and therefore the components of variance estimate from the analysis of variance and Grubbs' covariance estimate of product variance are identical.

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[2] Bennett, C. A. and Franklin, N. L., *Statistical Analysis in Chemistry and the Chemical Industry*. New York: John Wiley and Sons, 1954.

[3] Grubbs, F. E., "On estimating precision of measuring instruments and product variability," *Journal of the American Statistical Association*, 43 (1948), 243–64.

THE ECONOMICS OF THE PRESIDENT'S ECONOMIC REPORTS*

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THE *Economic Reports of the President* have, over the years, taken on a significance far beyond what might be anticipated from the Employment Act of 1946, which called for their annual presentation to Congress. The basic purpose of the Act was to establish the Federal Government's responsibility for the maintenance of full employment and maximum output. The *Reports* might, accordingly, have been largely restricted to reviewing the nation's experience in these respects, to outlining employment prospects, and to recommending policies designed to cope with actual or potential unemployment. In fact, the *Reports* have done much more than this; from the very beginning to a limited extent, but much more significantly in recent years, they have been full-scale reviews of the economic situation of the country. Their appraisals and policy recommendations have come to include such varied subjects as education, social security, agricultural policy, minimum wages, and even water pollution; tax problems essentially unrelated to the goal of full employment have achieved an emphasis at least equal to that given fiscal policies directly related to that goal; credit policies have been recommended for purposes quite remote from the maintenance of over-all economic stability.

It is impossible in a paper of this scope to review the whole range of problems and policies which the *Reports* have discussed. The following treatment will rather be confined to a general appraisal of the 1954, 1955, and 1956 *Reports*, and to the discussion of a few areas in which the analysis of the *Reports* appears to be vulnerable or the policy recommendations weak.

For the purposes of this review, the broad policy tenets of the administration will be accepted. These tenets are most clearly stated in the 1955 *Economic Report* (p. 2). They include the beliefs that "competitive markets, rather than governmental directives, are as a rule the most efficient instruments for organizing production and consumption," and that a free economy has a great capacity to generate jobs and incomes if an atmosphere of confidence is maintained. They envision a limited range of Federal government activities, concentrated on encouraging private initiative and curbing monopolistic tendencies, on restraining tendencies to inflation and recession, on providing essential public facilities, on widening opportunities for less fortunate citizens, and on helping individuals cope with the hazards of unemployment, illness, old age, and blighted neighborhoods.

GENERAL APPRAISAL

It is indeed difficult to find in the *Reports* policy recommendations which clearly conflict with the tenets presented above. With one glaring exception, the Soil Bank program, which will be discussed separately below, the major policies advocated in the *Reports* would bring our economy closer to the ideal

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implied in the tenets. Let us consider the steps which the *Reports* have advocated to bring the economy closer to the ideal of competitive resource allocation. The anti-monopoly program of the Administration is lauded; the minimum wage law is justified not as attacking the fundamental causes of poverty but as assisting "the comparatively small number of workers who are at the fringes of competitive labor markets"; continued extensive regulation of the railroad industry is questioned in the light of the increased effectiveness of competition from other forms of transportation; the maintenance of agricultural price supports at 90 per cent of parity is opposed as inconsistent with competitive resource allocation; and lower tariffs and liberalized customs procedures are called for.

The policies recommended in the *Reports* typically call for gradual rather than abrupt changes. Tariffs are to be lowered by small stages; flexible price supports are advocated to ease the transition to a competitive equilibrium; excise taxes are reduced but not eliminated. The 1954 *Report* boldly faces the controversial question of the corporation income tax, and concludes that "our ultimate goal should be a substantial equality of treatment of interest on debt and dividends on stocks," (p. 79) yet recommends only a modest dividend credit on the personal income tax, rather than a program for the gradual or abrupt elimination of the corporation income tax itself.

Some areas of clear divergence from the goal of competitive resource allocation are not mentioned at all in the *Reports*. These include the indirect subsidy to owner-occupied housing provided by our personal income tax law, and the incentive to commit real capital to uses in which its fruits can acquire special tax treatment as capital gains, or through percentage depletion.

The *Reports* thus characteristically recommend actions which would bring our economy gradually closer to the ideal to which the administration subscribes. They sometimes refrain from advocating action even though action is warranted in terms of their goals; they often advocate actions which proceed in the right direction but only by small steps. They do not, characteristically, move boldly; but on the other hand they rarely advocate a move in the wrong direction.

THE SOIL BANK PROGRAM

In the light of the general cautiousness of the *Reports*, it is indeed surprising that the boldest move they advocate is a move in the wrong direction. The Nation is making the best use of its resources when each resource is allocated to its most productive use. There can be little doubt that the most productive use of farm land is in the raising of agricultural commodities, and there can be no doubt at all that this use of the land is preferable to the land's lying idle. Yet the Soil Bank program calls for the elimination of substantial quantities of farm land from productive use, and for the expenditure of large sums of public funds to bring about this result.

The Soil Bank program arose from the simple fact that if competive forces alone had been allowed to set the prices of agricultural products, the incomes of farmers would have fallen even farther below their wartime and post-war peaks than in fact they did. The reduction in farm income levels is the com-

bined result of advances in agricultural technology, of the postwar recovery of European agriculture, and of the low price and income elasticities of demand for agricultural products. The competitive market's solution to the low-farm-income problem is simple enough; the low incomes in farming, as compared with other occupations, are themselves the incentive for a reduction in the number of farmers. Farmers would be induced to move out of farming until those who remained were receiving sufficiently high incomes that they had no incentive to move. The outflow of farmers would operate in two ways to give those who remain higher incomes: by reducing the supply of agricultural products, thus raising the market prices of those products; and by reducing the amount of manpower in agriculture as compared with land and other productive agents, thus raising the productivity of farm labor.

It would be hard to design a proposal more adverse to the workings of the competitive market mechanism than the Soil Bank program. Under it, the economy as a whole would lose the net product of the land removed from cultivation, while the farmers, being forced to work with less land, would have their own net productivity reduced. They would be compensated by the rise in agricultural prices resulting from this dual inefficiency, and by the substantial payments they would receive from the public till. Their higher income levels would, of course, impede their movement to more productive employments outside of farming.

In view of the boldness—almost effrontery—of the Soil Bank proposal, one cannot attribute its defects to the conservatism which pervades most of the other policy proposals in the *Reports*, and one may fruitfully inquire whether other programs, equally bold, might not be more suited to cope with our present agricultural problems.

Let us begin with the incentive, provided by the competitive market mechanism, for farmers to move out of agriculture. If the response to this incentive were strong and immediate, there would be no need for special policies to alleviate distress within agriculture. Farmers would move rapidly out until there was a substantial equality of incomes in farm and non-farm employments. Suppose, however, that it would take a large emigration of farmers from agriculture to achieve income equality, and that the rate at which farmers respond to the inducement to move out is rather slow. Under these circumstances farmers will for some time suffer lower incomes, and the question arises whether special programs are called for.

From the standpoint of the economic welfare of the rest of the society, the answer is no. The worker who is earning $50 a week in agriculture and who could earn $80 a week in industry will, if he makes the move, collect the full value of the extra product which his efforts produce. Any bribes or premiums paid to him, either to make the move or to alleviate his misery within agriculture, represent net deductions from the level of living of other members of the society. They are, in short, pure charity.

Let us assume, however, that other members of the society do indeed commiserate with the plight of the farmer, and are ready to suffer reductions in their real income in order to help him. How can this help best be given? If the farmer were paid a subsidy of $30 per week, he would never move out of

agriculture into more productive employment. A subsidy of less than $30 a week would help to alleviate his situation but would make him less likely to move than no subsidy at all. On the other hand, a subsidy of $30 per week to the farmers who abandon agriculture for other pursuits will stimulate the movement of workers out of agriculture and will leave those who remain better off. Thirty dollars a week of "charity" given in this form will clearly be of more net benefit to the farmer class and to the community as a whole than $30 a week given to farmers who stay on the land. Furthermore, while payments made to farmers on the land tend to keep them on the land and needful of further payments, subsidies to farmers who move off the land actually strike at the source of farm poverty. Indeed, probably the best single way to deal with farm poverty would be to offer a lump sum payment of, say, $1000 or $1500 to bona fide farmers who leave agriculture.

Price supports at 90 per cent of parity are analogous, in our example, to payments of $30 a week to farmers staying on the farm, and accordingly are an extremely ill-advised way of dealing with the structural changes which have struck farmers adversely. Flexible price supports would be analogous to payments of less than $30 a week to farmers staying on the land, with the amount of the payment declining over time. They are less costly than rigid 90 per cent supports, and they do not so strongly deter the movement of farmers off the land. However, flexible supports do slow down this movement, rather than speed it up. They are therefore a far from desirable way of coping with agricultural distress, though they may be quite appropriate for moderating the amplitude of cyclical swings in farm incomes.

DISTRESSED AREAS

Distressed areas of chronic "spot" unemployment present a problem quite similar to that facing American agriculture. Like agriculture, these areas have been the victims of technological advance and structural change, but owing to the "stickiness" of industrial wages, the reduction in income which they suffer tends to be reflected in unemployment rather than in low incomes for all. From the standpoint of society as a whole this is unfortunate, for the product which the unemployed workers might have produced during their period of idleness is irrevocably lost.

As in the case of agriculture, the distressed areas problem would not exist if labor were freely and rapidly mobile; and in many instances the problem would also be eliminated if new capital were ready to move in quickly to absorb the pool of unemployed. The *Reports* should be commended for recognizing that unemployment compensation, while useful and desirable for coping with cyclical swings in business activity, is not a really suitable solution to the problem of unemployment in distressed areas. The 1956 *Report* proposes, in addition, the placing of government contracts, as far as possible, in such areas; the allowance of special tax amortization privileges on defense production facilities located there; the provision of easy housing credit; and the provision of Federal loans covering up to a quarter of the costs of "projects that promise to improve a community's long-run economic outlook but for which financing cannot be obtained on reasonable terms from private sources."

In evaluating the remedies for chronic "spot" unemployment, care must be taken to distinguish among several varieties of the disease. In the case of an isolated mining town whose ore bodies are running out, a transfer of population to other areas is clearly called for, and policies similar to those suggested above for the case of agricultural workers are indicated. In the case of a group of specialized workers whose skills have been rendered obsolete by the advance of technology, the probable necessity of an ultimate reduction in their customary standard of living must be recognized, and attempts should not be made to maintain them in their old lines of employment. However, it is likely that most cases of structural unemployment occur in areas which are suitable for many forms of industry, and involve mainly types of labor which will be about as productive in alternative employments as they formerly were in the declining industry. Here the problem is to accelerate the reemployment of the unemployed.

The steps outlined in the 1956 *Report* for dealing with distressed areas are in the right direction, but they are weak and tentative, and do not reflect the high social costs entailed in heavy unemployment, even when that unemployment is localized. The direct social costs of unemployment are the foregone product of the unemployed; but in addition the unemployed receive unemployment compensation at the expense of the rest of society. In an area experiencing an increment of 10,000 in average unemployment over a period of two years, added outlays on unemployment compensation alone are likely to amount to more than $10 million. A bold program for coping with unemployment in distressed areas would recognize the worthwhileness of spending up to at least $1000 per reemployed worker on incentive schemes to accelerate reemployment. Such schemes might work either in the direction of accelerating the movement of workers to other areas or in the direction of attracting new capital into the distressed region. Of the programs suggested in the 1956 *Report*, only that calling for special accelerated amortization provisions on defense facilities and that calling for Federal loans covering part of the cost of new capital equipment entail any substantial monetary incentive toward the reemployment of the unemployed in new permanent jobs. Even apart from the probably-small magnitude of the incentives involved, both of these schemes tend to attract those industries which use the most capital per worker, instead of those which, with given capital, are likely to reemploy the most workers. The 1955 *Report* has made a tentative start on the distressed areas problem, and it is to be hoped that future *Reports* will come forward with proposals commensurate with the seriousness of this problem.

CYCLICAL UNEMPLOYMENT

In the field of monetary and fiscal policy, the *Reports* advocate the use of a whole arsenal of weapons: open-market operations, rediscount rate changes, changes in regulations governing the terms of particular types of credit, built-in fiscal stabilizers, changes in tax rates, and contracyclical regulation of government investment activities. To the extent that priority ratings among these weapons can be inferred from the *Reports*, they appear to be as follows. Built-in stabilizers, which require no policy decisions for their implementation, are the

first line of defense; second come monetary and credit policies, which can be implemented quickly and on administrative authority; third come tax rate changes and changes in government investment, which require considerable time to be put into effect.

The strong reliance placed upon built-in stabilizers is nowhere more evident than in the following analysis of the 1953–54 recession, taken from the 1955 *Report*:

> Between July 1953 and July 1954 total personal income derived from production decreased at an annual rate of 4.4 billion dollars. In the meantime, unemployment insurance and other social security payments to the public increased at a rate of 2.2 billion dollars, while tax payments by the public—quite apart from the change in rates that became effective in January 1954—fell at the rate of another billion. These two factors alone served, in very large part, to offset the over-all decline of personal income from production, and their effects were augmented by the operations of the farm price-support system.
>
> The experience of corporations was similar to that of individuals. While corporate income decreased at an annual rate of 7.4 billion dollars between the second quarter of 1953 and the second quarter of 1954, the tax liability of corporations was cut by 4.5 billion dollars merely as a result of the decline in income and quite apart from any change in the tax law. Once again, therefore, our taxing machinery automatically cushioned the impact of a declining income on the sums available to corporations for paying dividends or adding to their assets. (pp. 18–19)

The built-in stabilizers operated strongly in 1953–54, and they will operate still more strongly in the future if the increases in unemployment compensation rates which the *Reports* recommend are put into effect. However, there is some reason to believe that the *Reports* exaggerate the power of the stabilizers. The normal growth potential for output and incomes in our economy is about three per cent per year. Hence, although personal income derived from production fell, by mid-1954, only two per cent below its mid-1953 peak, it was five per cent below its potential; and although gross national product fell only four per cent over the period, it was by mid-1954 seven per cent, or close to 25 billion dollars, below its potential. The *Reports* may thus have set their sights too low in assessing the power and efficacy of stabilization policies.

Even when allowance is made for potential economic growth, however, it remains true that our built-in stabilizers work powerfully to limit the extent to which reductions in demand for the output of particular sectors of the economy get transmitted to other sectors. The more powerful these stabilizers are made, the more it will be true that such cyclical unemployment as appears in our economy will be concentrated in those industries of volatile demand (investment goods, construction, consumer durables) where recessions typically have originated. Monetary and credit policies, the *Reports'* second line of defense against the business cycle, exert a stronger influence on demand for long-lived assets than for short-lived assets, and hence can be fruitfully used to damp down the inherent volatility of the durable-goods sectors. General monetary policies will be most effective in situations in which most durable-goods sectors are suffering simultaneous declines, and least effective in situations in which unemployment is concentrated in one or a few durable-goods sectors. Credit policies striking particular sectors, such as automobiles or

residential construction, may be useful in smoothing out the fluctuations that would otherwise occur in those sectors, but may also induce fluctuations which would otherwise not occur. There is some evidence that our rapid recovery from the 1953–54 recession was bought in part as the price of a probable "over-expansion" of demand in the automobile and housing sectors, which was stimulated by particularly easy credit terms available to purchasers of cars and houses.

The 1956 *Report*, in calling for renewed Federal Reserve authority over the terms of installment credit and additional flexibility for government agencies in influencing mortgage terms, appears to be in accord with the foregoing line of analysis. However, the *Report* does not discuss the difficult problems involved in choosing the combination of general and specific policies most appropriate to a given situation.

The choice of policies to combat unemployment should depend on the distribution of the unemployment among industries, on the length of time the unemployment is expected to last, and on the mobility of labor out of the affected industries. If decline should strike an industry employing mainly labor of generalized skills, and located in areas where many other industries are also present, any policy which maintains a high level of effective demand in the economy as a whole will succeed in rapidly reemploying the unemployed. If, on the other hand, decline should strike an industry whose labor is highly specialized, or which is located, like the automobile industry, mainly in a single area, general stimuli to effective demand will not rapidly eliminate the unemployment. Specific stimuli of sufficient strength will do the trick, but their application entails the risk of artificially sustaining industries which are really undergoing a long-term decline. In general, if credit and other policies are to be used to smooth out fluctuations in demand in particular sectors of the economy, the policy-makers must know when demand is abnormally high and when it is abnormally low. Economists are not noted for their capacity to call the turns of economic development with great precision and accuracy. This should warn us against heavy reliance on policies aimed at stabilizing specific industries, though it should neither preclude their use in glaring instances nor slacken the efforts of economists, in and out of government, to bring their knowledge to the point where subtle and delicate stabilization policies aimed at specific trouble points will help more than they hurt.

ANALYTICAL GRADUATION OF FERTILITY RATES

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THE fertility ratio $f(y)$ is defined by

$$f(y) = \frac{k_y}{L_y} \tag{1}$$

where k_y denotes the number of live-born children, the age of the mothers on the day of delivery being y or more, but less than $(y+1)$ years, and L_y the number of observed women exactly y years old. The value of k_y can be established from vital statistics only, but it requires much work and time to do so. Besides, in many countries vital statistics are so defective as to be virtually useless. That is why only a few countries have completed fertility tables.

Giorgio Mortara[1] has, however, shown how the ratios of fertility can be computed easily from the results of population censuses according to the formula

$$f(y) = \frac{K_{y+1}}{L_{y+1}} - \frac{K_y}{L_y} = F(y+1) - F(y). \tag{2}$$

In this formula, L_y denotes the number of women $(y-0.5)$ or more, but less than $(y+0.5)$ years old on the day of the census, and K_y the total number of live children born to these women up to the census date. The Mortara formula can be easily derived in the following simple way.

We split K_{y+1}, i.e. the total number of live born children, into groups according to the age of the mothers on the day of delivery

$$k_\alpha + k_{\alpha+1} + \cdots k_{y-1} + k_y = K_{y+1} \tag{3}$$

where α stands for the lowest fertile age. Dividing (3) by L_{y+1} and taking (1) and (2) into consideration we get

$$f(\alpha) + f(\alpha+1) + \cdots f(y-1) + f(y) = F(y+1) \tag{4}$$

and consequently

$$f'(\alpha) + f'(\alpha+1) \cdots f'(y-1) = F(y). \tag{5}$$

From (4) and (5) follows

$$f(y) + \sum_{t=0}^{y-\alpha-1} [f(\alpha+t) - f'(\alpha+t)] = F(y+1) - F(y). \tag{6}$$

If $f(\alpha+t) = f'(\alpha+t)$, i.e., if the fertility rates of the generations L_y and L_{y+1} are equal (which corresponds more or less to reality, because fertility does not change much from one generation to the subsequent generation), then (6) is identically equal to (2), derived by Mortara by another procedure.

[1] *Methods of Using Census Statistics for the Calculation of Life Tables and Other Demographic Measures*, United Nations, Department of Social Affairs, Population Studies No. 7, Lake Success, New York, November 1949.

Mortara's formula (2) gives exact rates for the population covered by the census. For the total population, including persons who emigrated or died before the date of the census, the formula provides approximate figures only, but the error is so small that it may be disregarded.[2]

The practical use of formula (2), however, meets with a difficulty of mathematical nature. This formula consists of the difference between two quotients relating to two consecutive years of age, y and $(y+1)$. Hence even minute errors in $F(y)$ lead to a considerable discontinuity in the series of $f(y)$. The empirical values of the cumulative fertility function $F(y)$ must, therefore, be precisely graduated beforehand. Neither mechanical nor graphical methods of graduation give the necessary precision, so analytical methods of graduation must be used. Therefore we are still confronted with the task of finding an analytical function that represents the values of the cumulative fertility function $F(y)$ with sufficient accuracy.

Let us denote the lowest fertile age by α and the highest fertile one by ω. The graduation might be carried out with the function

$$\Phi(y) = \frac{F(y)}{F(\omega)}. \tag{7}$$

Therefore

$$\Phi(\alpha) = 0 \tag{8}$$

and

$$\Phi(\omega) = 1. \tag{9}$$

Obviously, it is necessary that

$$\frac{d\Phi(\alpha)}{dy} = 0 \tag{10}$$

and

$$\frac{d\Phi(\omega)}{dy} = 0. \tag{11}$$

The following analytical function meets conditions (7) to (11):

$$\Phi_n = e^{a_1\eta + \sum_{\nu=2}^{n} a_\nu(\omega-y)^\nu} \tag{12}$$

in which

$$\eta = \frac{\omega - y}{\omega - \alpha} + ln\frac{y - \alpha}{\omega - \alpha} \tag{13}$$

The values of the function η for $\alpha = 15$ and $\omega = 50$ are set forth in Table 1.

[2] Ivo Lah: *A Contribution to the Calculation of Fertility Tables on the Basis of Population Census*, World Population Conference, Rome, 1954, Meeting No. 5.

TABLE 1

y	η
15	$-\infty$
16	−2.583 9194
17	−1.919 3437
18	−1.542 4499
19	−1.283 3393
20	−1.088 7672
21	−0.935 0170
22	−0.809 4379
23	−0.704 4778
24	−0.615 2663
25	−0.538 4772
26	−0.471 7384
27	−0.413 2985
28	−0.361 8271
29	−0.316 2907
30	−0.275 8691
31	−0.239 9021
32	−0.207 8490
33	−0.179 2619
34	−0.153 7661
35	−0.131 0443
36	−0.110 8255
37	−0.092 8769
38	−0.076 9967
39	−0.063 0085
40	−0.050 7579
41	−0.040 1086
42	−0.030 9396
43	−0.023 1435
44	−0.016 6235
45	−0.011 2934
46	−0.007 0750
47	−0.003 8977
48	−0.001 6976
49	−0.000 4161
50	0.000 0000

The values $(\omega-y)^r=\xi^r$ may be found in various power tables of positive integers so that we will not quote them here.

Note that equation (10) is subject to the condition $a_1>1$, which is practically always fulfilled. (See (20).)

Let us determine the unknowns $a_1, a_2, \cdots, a_n$ by the method of least squares, i.e.

$$\sum_{y=x+1}^{\omega-1} (\Phi_n - \Phi_0)^2 = \text{minimum} \qquad (14)$$

where Φ_0 denotes the empirical values obtained in a population census. The graduation calculus is to be effected with the logarithms of functions Φ_n and Φ_0. The function $ln\ \Phi_n$ is to be expanded into the Taylor's series

$$ln\ \Phi_n = ln\ \Phi_0 + \frac{\Phi_n - \Phi_0}{\Phi_0} + R. \tag{15}$$

Assuming that the remainder R can be neglected in the calculation, we get from (15)

$$\sum(\Phi_n - \Phi_0)^2 = \sum \Phi_0^2(ln\ \Phi_n - ln\ \Phi_0)^2 = \text{minimum} \tag{16}$$

or

$$\sum \Phi_0^2(a_1\eta + a_2\xi^2 + \cdots a_n\xi^n - ln\ \Phi_0)\,\delta\ ln\ \Phi_n = 0. \tag{17}$$

From (17), it follows that the following equation system is necessary and sufficient for the determination of the unknowns $a_1, a_2, \cdots, a_n$:

$$\left.\begin{aligned}
a_1[\Phi_0^2\eta^2] + a_2[\Phi_0^2\eta\xi^2] + \cdots a_n[\Phi_0^2\eta\xi^n] &= [\Phi_0^2\eta\ ln\ \Phi_0] \\
a_1[\Phi_0^2\eta\xi^2] + a_2[\Phi_0^2\xi^4] + \cdots a_n[\Phi_0^2\xi^{n+2}] &= [\Phi_0^2\xi^2\ ln\ \Phi_0] \\
a_1[\Phi_0^2\eta\xi^3] + a_2[\Phi_0^2\xi^5] + \cdots a_n[\Phi_0^2\xi^{n+3}] &= [\Phi_0^2\xi^3\ ln\ \Phi_0] \\
\cdots\cdots\cdots

$$\left.\begin{aligned} F_1 &= 6.451e^{2.477\gamma} \\ F_2 &= 6.451e^{1.957\gamma-0.0005182\xi^2} \\ F_3 &= 6.451e^{2.581\gamma-0.0002187\xi^2+0.000001402\xi^3} \\ F_4 &= 6.451e^{3.371\gamma+0.0003586\xi^2-0.000014151\xi^3+0.000001324\xi^4} \end{aligned}\right\} \quad (20)$$

TABLE 2

ANALYTICAL GRADUATION OF FERTILITY RATES OF WOMEN ACCORDING TO THE CENSUS OF 1 SEPTEMBER 1940 IN BRAZIL

y	F_0	Φ_0	f_0	f_M	f_1	f_2	f_3	f_4
1	2	3	4	5	6	7	8	9
16	0.021	0.0033	4.0%	3.5%	4.5%	6.2%	4.9%	3.8%
17	0.061	0.0095	7.7	7.4	8.5	10.2	9.3	8.7
18	0.138	0.0214	12.4	12.8	12.7	14.1	13.7	13.8
19	0.262	0.0406	20.3	16.9	16.6	17.6	17.8	18.5
20	0.465	0.0721	21.9	20.5	20.2	20.5	21.1	22.1
21	0.684	0.1060	23.0	23.6	23.2	23.1	24.0	24.9
22	0.914	0.1417	26.0	26.2	25.8	25.2	26.2	27.0
23	1.174	0.1820	27.0	28.3	27.8	27.0	27.8	28.3
24	1.444	0.2238	32.0	29.9	29.5	28.2	29.1	29.2
25	1.764	0.2734	31.4	31.0	30.5	29.3	29.9	29.6
26	2.078	0.3221	27.9	31.6	31.3	29.8	30.2	29.7
27	2.357	0.3654	31.4	31.1	31.5	30.1	30.4	29.7
28	2.671	0.4140	29.4	30.5	31.4	30.1	30.0	29.5
29	2.965	0.4596	28.1	29.8	31.0	29.8	29.6	29.0
30	3.246	0.5032	27.3	29.0	30.4	29.3	28.9	28.4
31	3.519	0.5455	29.8	28.1	29.4	28.6	28.0	27.8
32	3.817	0.5917	30.3	27.1	28.3	27.6	27.1	26.9
33	4.120	0.6387	21.8	26.0	27.0	26.6	25.8	26.0
34	4.338	0.6725	20.3	24.8	25.5	25.2	24.7	24.9
35	4.541	0.7039	28.0	23.5	24.0	23.9	23.4	23.8
36	4.821	0.7473	26.1	22.1	22.3	22.4	21.9	22.4
37	5.082	0.7878	18.4	20.6	20.6	20.9	20.5	21.1
38	5.266	0.8163	16.1	19.1	18.7	19.2	18.9	19.5
39	5.427	0.8413	7.9	17.5	17.1	17.5	17.5	17.9
40	5.506	0.8535	19.0	15.9	15.2	15.8	15.8	16.2
41	5.696	0.8830	25.8	14.3	13.4	14.0	14.2	14.4
42	5.954	0.9230	17.9	12.6	11.6	12.3	12.5	12.7
43	6.133	0.9507	15.7	10.9	10.0	10.5	10.8	10.7
44	6.290	0.9750	−3.1	9.2	8.2	8.8	9.2	9.0
45	6.259	0.9702	7.3	7.5	6.6	7.1	7.5	7.2
46	6.332	0.9816	16.6	5.7	5.0	5.5	5.9	5.4
47	6.498	1.0073	−0.7	3.9	3.5	3.8	4.1	3.7
48	6.491	1.0062	−0.6	2.1	2.0	2.3	2.5	2.2
49	6.485	1.0053	−3.4	0.3	0.7	0.7	0.8	0.6
50	6.451	1.0000						

With respect to continuity, there is nothing objectionable in the analytically graduated values f_1, f_2, f_3, f_4. In order to estimate the closeness of agreement between the graduated values and the empirical ones, we give the sum of the squared deviations for all the ages from 16 to 43 years:

$$\sum_{y=16}^{43} (f_1 - f_0)^2 = 484\%$$

$$\sum_{y=16}^{43} (f_2 - f_0)^2 = 458\%$$

$$\sum_{y=16}^{43} (f_M - f_0)^2 = 428\%$$

$$\sum_{y=16}^{43} (f_3 - f_0)^2 = 426\%$$

$$\sum_{y=16}^{43} (f_4 - f_0)^2 = 426\%$$

The deviations of f_1 and f_2 from f_0 are greater than the deviations of f_M, whereas the deviations of f_3 and f_4 are about the same as those of f_M. We shall therefore compare Mortara's values with our values f_3 and f_4 only.

Within the age interval from 29 to 49 years, Mortara's values f_M are about the same as f_3. Greater differences between f_M and f_3 or f_4 are found solely in the lowest fertile ages and in the ages of maximum fertility. It seems that Mortara with his graphic graduation obtained rather too low values in the lowest fertile ages. The sum of squared deviation from the crude values f_0 in the age interval from 16 to 20 years for f_M is of the order

$$\sum_{y=16}^{20} (f_M - f_0)^2 = 14\%$$

and for f_4 only

$$\sum_{y=16}^{20} (f_4 - f_0)^2 = 6\%$$

According to Mortara, the maximum fertility (age $y=26$ years) amounts to 31.6%, whereas from f_4 it is only 29.7%. Mortara's maximum seems thus to be too high. Mortara's graph of f_M in Figure 2, page 50, of his *Methods of Using Census Statistics* . . . is obviously too leptokurtic at the age $y=26$, whereas the graph of f_4 is platykurtic.

In our opinion, the values f_3 and f_4 within the age interval from 16 to 28 years obtained by analytical graduation correspond better to reality than the values f_M obtained by graphic graduation. Even if this were not the case, we would be in a position to compute by further analytical graduation still more exact values f_5, f_6, f_7, etc.

ASYMPTOTIC EFFICIENCIES OF A NONPARAMETRIC LIFE TEST FOR SMALLER PERCENTILES OF A GAMMA DISTRIBUTION

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This paper does not present new statistical methods. Rather, it shows that a well known nonparametric procedure which is valid under extremely general conditions also has high efficiency and other favorable properties for certain types of life testing situations. This nonparametric method is the sign test and its confidence interval and point estimate analogs. Sign test procedures and methods of applying them have already received extensive coverage in the literature (e.g., [2]). Consequently, in this paper emphasis is placed on fields of application instead of on methods of application. However, the basic methods for obtaining tests, confidence intervals, and point estimates of the sign test type are restated for convenience. The properties investigated are concerned with the estimation and testing of smaller population percentage points for the case of a sample from a gamma probability distribution. The investigations show that appropriate use of the sign test type of nonparametric tests and estimates sometimes can yield a saving in cost or time without loss of statistical efficiency, since the experiment can be stopped when only a fraction of the items being life tested have failed. The nonparametric results are found to be highly efficient compared to the "best" parametric results based on the same fraction of items failed for the case of small population percentage points; this conclusion appears to be valid for any sample size and for any reasonable type of statistical population.

INTRODUCTION AND DESCRIPTIVE OUTLINE

A DESCRIPTION of practical situations which are of a life testing type and an outline of the previous literature on this subject already have been given [3]. To save space, background information of this nature will not be repeated in this paper.

Here we consider a life testing situation where a number n of similar items are simultaneously life tested to determine their failure properties. The random variable for each item is the time to failure or some continuous strictly monotonic increasing function of time to failure. The n values obtained are considered to be a random sample from a continuous statistical population. The problem is to obtain information concerning a small percentage point of this population on the basis of the sample values.

A valuable property of the type of random variable occurring for life testing situations is that the r smallest values of a sample can be obtained without the necessity of determining the remaining n-r sample values. The first item to fail furnishes the value of the smallest order statistic of the sample, the second failing item determines the value of the next to smallest order statistic, etc. Thus statistical results based entirely on the r smallest values of the sample have the advantage that the experiment can be stopped when the rth failure occurs.

The investigations of this paper are concerned with two different types of problems, which are descriptively referred to as the Fraction Failed and All

Failed cases depending on whether part or all of the sample items must be failed. First let us consider the Fraction Failed case. This deals with the statistical efficiency (denoted by 100E%) of the nonparametric results as compared with the corresponding "best" (i.e., highest efficiency attainable) results based on the same number of items failed and the same sample size. For given n, a nonparametric result (significance test, confidence interval, or point estimate) of the type considered requires the determination of one or more order statistics. In life testing situations, this implies the failure of the items which determine all order statistics up to and including the largest of the order statistics needed for application of this nonparametric result. The comparison is between the nonparametric result and the corresponding "best" result based on all order statistics with values less than or equal to this largest order statistic.

Efficiency values are computed only for the situation of indefinitely large n and gamma probability distributions (see Derivation section of this paper for definition of gamma probability distributions). However, intuitive considerations indicate that these asymptotic values furnish lower bounds for the efficiencies of the nonparametric results for all values of n. For the Fraction Failed case, efficiency lower bounds are evaluated which appear to be valid for all reasonable types of statistical populations (including all types of gamma probability distributions). These lower bounds indicate that the nonparametric results have rather high efficiencies for the Fraction Failed case at all sample sizes and for virtually all statistical populations of practical importance.

Another quantity F is also computed for the Fraction Failed case. The factor F has the property that the nonparametric results based on a sample of size nF (nearest integer) furnish at least as much "information" (i.e., have properties at least as favorable) as the corresponding "best" results based on a sample of size n. Thus F equals the reciprocal of the lower bound efficiency value. The fact that nF exceeds n allows results which are not "best" for sample size nF to have more favorable properties than the "best" results for sample size n. If nr items are failed for the nonparametric result and sample size nF, then the corresponding "best" result and sample size n is based on the nr/F (to nearest integer) smallest order statistics of the sample of size n. Thus, for the Fraction Failed case and smaller population percentage points, a small sample size increase and use of the nonparametric results appears to be at least as desirable as use of the "best" possible results based on a sample of the original size from an "information" furnished viewpoint.

Now let us consider the All Failed case. Then the comparison is between a nonparametric result and the corresponding "best" result based on the same sample size for the situation where all the items of a sample are failed. Although derived on an efficiency basis, the material presented for the All Failed case does not directly deal with efficiencies but rather involves the determination of values for the factor F. For the All Failed case, the factor F has the property that the nonparametric results based on a sample of size nF furnish at least as much "information" as the corresponding "best" results based on all values of a sample of size n. Efficiency lower bounds for the nonparametric results can be obtained as reciprocals of the corresponding F values.

The values of F are obtained by investigating the large sample situation for gamma probability distributions. Intuitive considerations and experience with nonparametric results of the type used indicate that F increases with n; thus the asymptotic ($n \to \infty$) value for F is the one sought. Motivation for considering random variables with gamma probability distributions arises from the fact that this type of random variable and transformations of it with the same asymptotic efficiency properties appear capable of approximating most continuous life test situations of practical interest.

The All Failed case differs from the Fraction Failed case in that the value obtained for F depends strongly on the particular gamma probability distribution used. The different gamma distributions are identified by the value of a known parameter λ. Thus, the value of F depends on the value of λ. In applying the procedures which use F as a basis, the value of λ is assumed to be known or have known bounds which are not too far apart. This approximate information about λ usually must be obtained from technical considerations concerning the probability process encountered or from past experience with situations of a similar nature.

By starting with a larger number of items but stopping the experiment when a small fraction have failed, the same "information" sometimes can be obtained with less expense and/or experiment time than if fewer items were used and the experiment continued until all failed. In particular, use of the nonparametric results based on a sample of nF items may save cost and/or time compared to the corresponding "best" results based on all the values of a sample of n items. Rough knowledge of λ and the corresponding upper bound for F is sufficient for obtaining nonparametric results which furnish at least as much "information" as the corresponding "best" results for this situation.

Having established values for λ and F, the question arises as to whether use of the resulting nonparametric procedures will actually be preferable. In the most general form, this represents a complicated problem. However, the two principal ways of accomplishing a saving are easily specified. First, the number of items failed may be reduced by using the nonparametric results rather than those based on all the sample values. Such a reduction will always result in shortening the time for the experiment. However, even when more items must be failed by using the nonparametric results, experiment time still may be shortened. Determination of the maximum order statistic needed for the case of the nonparametric results with sample size nF directly determines the number of items failed for the nonparametric result while n is the number of items failed for the corresponding "best" results. As an aid in deciding how much time is saved (or lost), a method is presented for estimating the relative amount of experiment time for the nonparametric results and sample size nF as compared to the corresponding "best" results for all items failed and sample size n.

The procedure of reducing the number of items failed results in a saving if the cost associated with a failed item is much larger than the cost for a life tested item which did not fail. Reducing the time required for the experiment can be important if the results are needed as soon as possible, if the equipment

being used is needed for some other purpose, or if the experiment is such that the cost of operation increases with time. Jacobson [5] has called attention to advantages of using results based on the r smallest order statistics for life testing situations.

Now let us define the well known nonparametric results whose properties are considered in this paper. The notation $t(1), t(2), \cdots, t(n)$ is used to represent the sample values arranged in order of increasing algebraic value. Thus $t(1)$ is the smallest order statistic of the sample, $t(2)$ is the next to smallest order statistic, etc. Also the notation θ_q is used for the $100q$ percentage point (assumed unique) of the statistical population from which the sample was drawn. Then the one-sided and two-sided confidence intervals for θ_q are defined by the relations

$$Pr[t(r) < \theta_q] = \sum_{i=r}^{n} \frac{n!}{i!(n-i)!} q^i (1-q)^{n-i},$$

$$Pr[t(s) > \theta_q] = 1 - Pr[t(s) < \theta_q],$$

$$Pr[t(s) < \theta_q < t(r); s < r] = 1 - Pr[t(r) < \theta_q] - Pr[t(s) > \theta_q].$$

The significance tests considered are those obtained from these confidence intervals in the usual manner. These tests are equivalent to sign tests. A single order statistic is used as a point estimate of θ_q. In this paper the order statistic $t([nq+\frac{1}{2}])$ is advocated for this purpose when $nq \geqq \frac{1}{2}$ ($[nq+\frac{1}{2}]$ denotes the largest integer $\leqq nq+\frac{1}{2}$). As an example, let $n=7$, $q=.4$, $t(1)=6.215$, $t(2)=8.333$, $t(3)=8.349$, $t(4)=9.767$, $t(5)=10.113$, $t(6)=11.998$, $t(7)=14.318$. Then $Pr(14.318<\theta_{.4})=.0016$; (6.215, 14.318) is a two-sided confidence interval for $\theta_{.4}$ with a confidence coefficient of .970; a one-sided significance test for comparing $\theta_{.4}$ with a specified value $\theta_{.4}^{(0)}$ at the .028 significance level is

$$\textit{Accept that } \theta_{.4} > \theta_{.4}^{(0)} \textit{ if } 6.215 > \theta_{.4}^{(0)};$$

and 8.349 is a point estimate for $\theta_{.4}$.

The word "efficiency" occurs throughout the paper. This term is used in comparing the knowledge to be obtained from the nonparametric results with that obtainable from the corresponding optimum results for the particular situation being considered. An efficiency of 100E% implies that, for the situation considered, the nonparametric results based on n/E sample values furnish the same "information" as the corresponding optimum results based on n sample values. A method of numerically evaluating an efficiency is difficult to specify except for the case of large n. Fortunately, consideration of asymptotic ($n \to \infty$) efficiencies appears to be sufficient for the situations analyzed in this paper. As the sample size increases, a single sample value (i.e., order statistic) would seem to become relatively less important when compared to the totality of all smaller or equal sample values and when compared to the totality of all sample values. These intuitive considerations strongly indicate the lower bound for E and the corresponding value for F are obtained for the asymptotic case. Experience with nonparametric results of the type considered in this paper tends to bear out this contention (see, e.g., [7]). A detailed definition of "asymptotic efficiency" along with a discussion of its properties is given in [8].

The next section of the paper calls attention to some appropriate fields of applications for the nonparametric results and emphasizes the wide range of validity of these results; also approximate validity of the lower bound for E (and the value obtained for F) for virtually all continuous type life testing populations is discussed and intuitively justified for the Fraction Failed case. The following section is titled Results. Here the situations investigated are described in detail, the results obtained are presented, and procedures for practical application of these results are outlined. The final section is called Derivations. This section contains an outline of the mathematical foundations for the results stated in preceding sections of the paper.

VALIDITY AND FIELDS OF APPLICATION

Use of the nonparametric results is especially desirable for the case of significance tests. Then an approximate upper bound for the time duration of the experiment can be determined in advance. That is, the specified hypothetical value for θ_q furnishes an upper bound for the values of the variable which need to be considered. For example, if time to failure is the variable and the hypothetical value for θ_q is 36 hours, the duration of the experiment will not exceed 36 hours. The time for the experiment will be less than the upper bound determined on this basis if the order statistic, or statistics, used for the nonparametric test turn out to be less than the value specified for θ_q.

A class of significance tests which is worthy of special mention for life testing situations is single sample acceptance inspection plans based on attributes. Here a small population percentage point usually is investigated so that the considerations of the paper are applicable. Also for this case acceptance inspection by attributes is equivalent to using a nonparametric test of the type considered in this paper. Thus single sample acceptance inspection by attributes has the favorable properties outlined in this paper for the nonparametric significance tests. This is one instance where acceptance inspection by attributes may furnish more "information" per unit cost than acceptance inspection by variables using the standard procedures.

If care is exercised in setting up the experiment, the requirements of a continuous variable and statistical independence can nearly always be satisfied to a reasonable approximation. However, the condition that all observations are drawn from the same statistical population is more difficult to satisfy and often not easy to verify on intuitive grounds. The nonparametric results avoid this difficulty by allowing a weakening of this condition. For the validity of the nonparametric results, it is sufficient that the separate probability distributions for the items life tested have the same $100q$ percentage point, irrespective of their shapes. If the value of n is at least moderately large, even the requirement of a common $100q$ percentage point can be noticeably weakened. Let θ_q have the property that $Pr(t_i<\theta_q)=q_i$ for the ith population $(i=1, \cdots, n)$ and $q=\sum_1^n q_i/n$. Thus θ_q is an "average" $100q$ percentage point for the n populations from which the observations were drawn. If the variation among the values of the q_i is not great, the nonparametric results for this "average" θ_q are approximately the same as for the case of a random sample (n large). This follows directly from the results of [6] and [9]: The significance level or confidence

coefficient for a nonparametric test or confidence interval can be evaluated by considering the probability distribution of the number of "successes" for n statistically independent binomial events with probabilities $q_1, \cdots, q_n$. For situations of the type approximated in practice, this probability distribution should be nearly of a normal and/or a Poisson form for large n. However, the material of [6] and [9] shows that for both forms the parameters involved are very nearly determined by the value of $q = \sum_1^n q_i/n$ if the value of $\sum_1^n (q_i - q)^2/n$ is of the same order of magnitude or smaller than q.

The nonparametric results remain valid under any continuous monotonic transformation of variable. This invariance property furnishes a basis for some considerations which indicate that the lower bound efficiency results (and corresponding F values) for the Fraction Failed case hold for virtually all continuous probability distributions which are of practical interest for life test situations. If the probability distribution of the observed continuous variable is of any reasonable type, there should exist at least one continuous monotonic change of variable which will transform this observed variable into a variable which has a gamma distribution. If such a continuous monotonic change of variable does exist, then the asymptotic efficiency lower bounds presented for the Fraction Failed case hold for the observed variable.

The decision of whether to use the nonparametric results rather than competing results is not as complicated for the Fraction Failed case as for the All Failed case. Experiment time is not a major consideration in the Fraction Failed case since the nonparametric results and any competing results with higher efficiencies should require about the same amount of experiment time. If competing results with higher efficiencies are not available for the situation encountered the decision is straightforward. If such results are available the uncertainty involved in the assumed form of the situation encountered must be weighed against the increase in the number of items tested and failed for the nonparametric results. If $q \leqq .25$ for the Fraction Failed case, the assumption uncertainty would have to be rather small and/or the cost for extra items tested and failed quite large if the competing results are to be preferred to the nonparametric results.

RESULTS

This section contains a statement of the computed asymptotic efficiencies and factors along with an explicit specification of the class of gamma distributions considered. Some discussions which emphasize the generality of the situations analyzed are presented. Also, for given λ and F, a method is outlined for help in estimating the relative amount of experiment time required for the nonparametric results as compared to the time required for corresponding results based on all items failed.

For a life testing situation, the observations have some minimum possible value which is finite. Here this minimum value is assumed to be known. The effect of this procedure is to decrease the efficiency of the nonparametric results (compared to situation where this value is unknown) and make the computed efficiency lower bounds more conservative. No generality is lost from an efficiency investigation viewpoint by taking this known minimum value to be

zero. In many cases the actual value of this lower limit is zero. Consequently, all the gamma distribution of the class considered have minimum value zero.

The family of gamma distributions selected for investigation depends on the values of two parameters. The probability density function for this class of distributions is defined by

$$f(t;\xi,\lambda)=\begin{cases}\dfrac{\xi^{-\lambda}t^{\lambda-1}}{\Gamma(\lambda)}e^{-t/\xi}, & \text{for } t\geqq 0\\ 0, & \text{for } t<0,\end{cases}$$

where the parameters ξ and λ are both positive. This family of probability distributions appears capable of approximating many continuous life test sampling situations of practical interest. As an example, for $\lambda=1$ the distribution is of the exponential type considered in [3]. For $\xi=2$ and λ an integral multiple of $\frac{1}{2}$, a χ^2 distribution is obtained. If λ is large, the distribution is approximately normal (tends to normality as $\lambda\to\infty$).

In determining asymptotic efficiencies and factors, the value of λ is assumed to be known. This increases the range of applicability of the results and greatly simplifies the computations. Since assuming λ known decreases the efficiency of the nonparametric tests and estimates (compared to the situation where λ also is unknown), the "lower bound" property of asymptotic efficiencies of nonparametric results is preserved. That is, efficiency values obtained by assuming an asymptotic situation with λ known should furnish lower bounds for the true efficiency situation with both ξ and λ unknown. Since the factor F is the reciprocal of the lower bound efficiency for the situation considered, the asymptotic case with λ known should furnish satisfactory values for F. The cases for which computations were made are $\lambda=.05, .1, .5, 1, 2, 5, 10, 20, 50$ and $q=.01, .02, .05, .1, .2, .3, .7$. The values for these cases should furnish a satisfactory indication of the asymptotic situation for $.05\leqq\lambda\leqq 50$ and $.01\leqq q\leqq .7$.

First, let us consider the asymptotic efficiency comparison of the nonparametric results with the corresponding "best" results based on the same percentage of items failed (i.e., the Fraction Failed case). Explicitly, if $t(r)$ is the largest order statistic needed in applying the nonparametric result considered, the comparison is with the corresponding "best" result based on $t(1), t(2), \cdots, t(r)$. The computed values (not listed to save space) show that, for fixed q, the asymptotic efficiency is a monotonically decreasing function of λ; this property seems to hold for all positive λ. The minimum asymptotic efficiency is obtained by considering the limiting situation when $\lambda\to\infty$.

As λ becomes infinite the gamma distribution tends to a normal distribution with mean $\lambda\xi$ and variance $\lambda\xi^2$. However, the asymptotic efficiency for this limiting case is at least as great as the asymptotic efficiency for the case of normality and known variance. Use of the normal distribution with known variance extends the application validity of the results and reduces the required computations. Table 1 contains the efficiency lower bound values and corresponding $F=F(q)$ values computed on this basis for $q=.01, .02, .05, .1, .2, .3, .5, .7$. The values of Table 1 should hold for any n if the population distribution is of the gamma or normal form (variance known or unknown, mean

TABLE 1

APPROXIMATE EFFICIENCY LOWER BOUNDS AND CORRESPONDING F VALUES FOR FRACTION FAILED CASE

q	.01	.02	.05	.1	.2	.3	.5	.7
$100E$ %	99	98	97	95	92	88	78	61
F	1.01	1.02	1.03	1.05	1.09	1.14	1.28	1.64

unknown). The validity discussion given in the preceding section indicates that the results of Table 1 should hold for virtually all continuous populations of a life testing type for the Fraction Failed case.

Now let us consider the results for the case where all the items of the sample are failed (the All Failed case). Here the probability distribution is of the gamma form and the value of λ is assumed to be specified by the experimenter (perhaps estimated from past experience with situations of a similar nature). For given q and λ, the problem is to determine the value of the factor F. By definition F equals the reciprocal of the lower bound on efficiency of the nonparametric results as compared to the "best" results for all items failed. As in the Fraction Failed case, the value for $F=F(q, \lambda)$ is obtained as the reciprocal of the asymptotic $(n\to\infty)$ efficiency of the nonparametric results for the situation investigated. Table 2 contains the resulting asymptotic values of F for $\lambda=.05, .1, .5, 1, 2, 5, 10, 20, 50$ and $q=.01, .02, .05, .1, .2, .3, .7$. These parameter values should cover the ranges of practical interest. Examination of Table 2 shows that, for fixed q, the upper bound value for F can change appreciably as λ varies from .05 to 50.

For specified q and λ, Table 2 yields a value of F with the property that the nonparametric results based on a sample of nF items furnishes at least as much "information" as the corresponding "best" results based on all the values of a sample of n items. If the nonparametric results based on nF items result in no over-all loss (compared to the alternative considered) when time, expense, etc., are taken into consideration, their use is recommended by their ease of application and general validity. In deciding on the desirability of the nonparametric results, based on a sample of nF items, the number of items failed, the size of nF, and the total time for the experiment are usually of primary importance. The number of items failed is readily determined on the basis of the maximum order statistic needed for the nonparametric result used. If nF is too large, the saving in time and reduction in items failed may be offset by the magnitude of the experiment. The problem of estimating the total experiment time for the nonparametric results and for results based on all items failed is more difficult. However, it is often possible to develop some rough guides for making these estimations.

By hypothesis the observed variable (denoted by t) is a continuous strictly monotonically increasing function of the time elapsed since beginning of the experiment (denoted by T). If this function is known and satisfies some rather general conditions, rough estimation of the relative total experiment time for a nonparametric result (nF items) compared to this time for the alternative possibility considered (n items, all failed) can be made. Let the known continuous monotonic function be denoted by $A(T)$; i.e., $t=A(T)$. Let $t(r; nF)$ represent the value of the maximum order statistic needed for the nonparamet-

TABLE 2

VALUES OF F WHEN $.05 \leq \lambda \leq 50$ AND $.01 \leq q \leq .7$ FOR ALL FAILED CASE

λ \ q	.01	.02	.05	.1	.2	.3	.7
.05	1980	980	380	180	80	46.5	8.5
.1	990	490	190	90	40	23	4.4
.5	198	99	38	18.2	8.4	5.2	1.82
1	101	50	20	10	5	3.4	1.62
2	54	28.5	12.4	6.5	3.6	2.6	1.56
5	31.2	17.1	7.9	4.6	2.8	2.2	1.57
10	24.2	13.7	6.6	4.0	2.5	2.0	1.60
20	21.2	12.0	6.1	3.8	2.4	2.0	1.72
50	18.7	11.3	5.8	3.7	2.4	2.0	1.83

ric result while $t(n; n)$ represents the maximum sample value for the All Failed case. Here $t(r; nF)$ is based on a sample of size nF and $t(n; n)$ is based on a sample of size n. Let T_r represent the total experiment time for the nonparametric result based on sample size nF while T_n represents the experiment time for the competitive result based on sample size n and all items failed. Then the values of T_r and T_n are determined from the relations.

$$A(T_r) = t(r; nF), A(T_n) = t(n; n),$$

which have unique solutions. Unfortunately, the values of $t(r; nF)$ and $t(n; n)$ cannot be known until the respective experiments have been completed. However, the relative amount of experiment time for the two methods can be estimated by determining a known function which furnishes a "representative" value of T_r given a corresponding "representative" value of T_n, and vice versa. If the "representative" value of T_r is sufficiently less than the corresponding "representative" value of T_n for all T_n values which are considered possible, the nonparametric results can be accepted as preferable on the basis of required experiment time.

The first step of the procedure adopted for defining "representative" values for T_r and T_n consists in determining values $\alpha = \alpha(\epsilon)$ and $\beta = \beta(\epsilon)$ such that

$$Pr[t(r; nF) > \theta_\alpha] = Pr[t(n; n) > \theta_\beta] = \epsilon,$$

where ϵ is appropriately chosen (e.g., $\epsilon = .1$ or $.05$). The determination of α and β is easily done from the relations

$$Pr[t(r; nF) > \theta_\alpha] = \sum_{i=0}^{r-1} \frac{(nF)!}{i!(nF - i)!} \alpha^i (1 - \alpha)^{nF-i} = \epsilon,$$

$$Pr[t(n; n) > \theta_\beta] = 1 - \beta^n = \epsilon.$$

Then the "representative" values of T_r and T_n (denoted by $\hat{T}_r$ and $\hat{T}_n$) are defined by the relations

$$A(\hat{T}_r) = \theta_\alpha, A(\hat{T}_n) = \theta_\beta.$$

These "representative" quantities have the property that

$$Pr(T_r > \hat{T}_r) = Pr[A(T_r) > A(\hat{T}_r)] = Pr[t(r; nF) > \theta_\alpha] = \epsilon,$$

$$Pr(T_n > \hat{T}_n) = Pr[A(T_n) > A(\hat{T}_n)] = Pr[t(n; n) > \theta_\beta] = \epsilon.$$

That is, the probability that the experiment time for the nonparametric results exceeds $\hat{T}_r$ is the same as the probability that this time for the competitive results with all items failed exceeds $\hat{T}_n$; moreover, this common value ϵ can have any desired value. Thus, with suitable selection of ϵ, the quantities $\hat{T}_r$ and $\hat{T}_n$ would seem to furnish satisfactory "representative" values for the experiment durations for the methods compared.

Since θ_α and θ_β depend on the unknown parameter ξ, the values of both $\hat{T}_r$ and $\hat{T}_n$ cannot be determined. However, the value of $\hat{T}_r$ corresponding to a specified value of $\hat{T}_n$ can be evaluated (similarly for $\hat{T}_n$ when $\hat{T}_r$ is specified). As shown in the Derivations section, $\theta_q = \xi\phi_q(\lambda)$, where $\phi_q(\lambda)$ is the $100q\%$ point of the standardized gamma type statistical population with probability density function given by equation (1) in the next section. In terms of the known quantity $\phi_q(\lambda)$,

$$A(\hat{T}_r)/A(\hat{T}_n) = \theta_\alpha/\theta_\beta = \phi_\alpha(\lambda)/\phi_\beta(\lambda).$$

This is the equation relating $\hat{T}_r$ and $\hat{T}_n$. Note that no experimental results are needed for this comparison between a "representative" value for the length of the experiment using the nonparametric results and sample size nF and a comparable "representative" value for the length of an experiment with sample size n and all items failed. Values of $\phi_q(\lambda)$ are listed in Table 3 for $\lambda = .05$, .1, .5, 1, 2, 5, 10, 20, 50 and $q = .01$, .02, .05, .1, .2, .3, .7, .9, .95, .99, .999, .9999, .99999. Use of Table 3 combined with interpolation should furnish the values of $\phi_\alpha(\lambda)$ and $\phi_\beta(\lambda)$ for virtually all λ, α, β of interest.

Even though the observed variable may not have an approximate gamma distribution, a continuous strictly monotonic change of variable may be known which will yield a transformed variable whose distribution is very nearly of the gamma type. Then, asymptotically, the efficiency properties of this transformed variable are the same as those for the observed variable. Consequently the upper bound values for the factor F associated with the observed variable are determined by the value of λ for the transformed variable and are those listed in Table 2. Having determined the value for F, the sample size and number of items failed for the nonparametric result considered for application are easily evaluated. A relationship between $\hat{T}_r$ and $\hat{T}_n$ can also be obtained if the function relating the observed variable τ to the transformed variable t and the function relating t to T are both known. Let these two relationships be denoted by

$$\tau = B(t), \qquad t = A(T),$$

so that $\tau = B[A(T)]$. The function $A(T)$ is necessarily increasing. This implies that $B(t)$ is also increasing; otherwise τ could not have a distribution which is very nearly of the gamma type. Thus, τ is a strictly monotonic increasing function of T and the analysis of the preceding paragraph implies that

$$B[A(\hat{T}_r)]/B[A(\hat{T}_n)] = \phi_\alpha(\lambda)/\phi_\beta(\lambda)$$

is a valid relationship between $\hat{T}_r$ and $\hat{T}_n$ for this situation which is expressed in terms of known quantities.

Finally, it might be pointed out that precise determination of λ is not required. A moderate deviation from the true value (say, at most 25%) does not have an appreciable effect on the value for F. Also the values used for F tend to be too large so that a moderate λ error usually can be absorbed.

Numerical Example. To illustrate the determination of a relationship between $\hat{T}_r$ and $\hat{T}_n$, an example will be analyzed in moderate detail. The case considered is such that the observed variable must be transformed in order to obtain a variable with a gamma probability distribution.

Let $n=100$, $q=.2$, $\lambda=2$, $\epsilon=.05$,

$$A(T) = 4 + 3T + T^2, \qquad B(t) = \sqrt{t}.$$

The nonparametric result considered is a one-sided confidence interval for $\theta_{.2}$ with the lower limit determined by the sample values and a confidence coefficient of .90. Then $F=3.6$ from Table 2 so that $nF=360$. Also

$$Pr[t(62) < \theta_{.2} < \infty\,;\, nF = 360] = .90$$

on the basis of the normal approximation to the binomial distribution. Thus $t(62)$ is the maximum order statistic used for the nonparametric confidence interval considered so that $r=62$. Again using the normal approximation to the binomial distribution, the solution of the equation

$$Pr[t(62; 360) > \theta_\alpha] = .05$$

is found to be $\alpha=.209$. The relation

$$\beta = (.95)^{1/100} = .9995$$

furnishes the value of β. Then from Table 3 and interpolation,

$$\phi_{.209}^{(2)} = .84, \qquad \phi_{.9995}^{(2)} = 13.$$

Consequently the relationship between $\hat{T}_r$ and $\hat{T}_n$ is

$$(4 + 3\hat{T}_n + \hat{T}_n^2)^{1/2} = 15.5(4 + 3\hat{T}_r + \hat{T}_r^2)^{1/2},$$

TABLE 3

VALUES OF $\phi_q(\lambda)$ FOR $.05 \leqq \lambda \leqq 50$ AND $.01 \leqq q \leqq .99999$

λ \ q	.01	.02	.05	.1	.2	.3	.7	.9	.95	.99	.999	.9999	.99999
.05	5.84 $\times 10^{-61}$	6.13 $\times 10^{-56}$	5.59 $\times 10^{-47}$	5.61 $\times 10^{-41}$	6.13 $\times 10^{-36}$	2.03 $\times 10^{-31}$	4.65 $\times 10^{-4}$	.076	.265	1.09	2.74	4.62	6.62
.1	6.07 $\times 10^{-31}$	6.21 $\times 10^{-28}$	5.92 $\times 10^{-24}$	6.07 $\times 10^{-21}$	6.21 $\times 10^{-18}$	3.58 $\times 10^{-6}$	.0175	.27	.58	1.59	3.36	5.31	7.35
.5	7.86 $\times 10^{-6}$	3.14 $\times 10^{-4}$	1.97 $\times 10^{-3}$	7.88 $\times 10^{-3}$	.032	.074	.54	1.35	1.92	3.32	5.40	7.56	9.75
1	.0161	.0202	.0513	.105	.223	.357	1.204	2.30	2.90	4.61	6.91	9.21	11.52
2	.149	.215	.353	.529	.823	1.095	2.44	3.88	4.75	6.63	9.22	11.75	15.2
5	1.28	1.53	1.97	2.43	3.09	3.63	5.89	7.95	8.95	11.6	14.7	17.7	20.6
10	4.13	4.62	5.43	6.22	7.29	8.13	11.39	14.2	15.7	18.8	22.7	26.0	29.5
20	11.08	11.92	13.26	14.53	16.19	17.44	22.08	25.8	27.8	31.9	36.7	41.0	48.1
50	35.07	36.57	38.96	41.18	43.97	46.06	53.45	58.0	61.6	68.1	74.4	80.6	86.1

or, by squaring both sides and solving the quadratic in $\widehat{T}_n$,

$$\widehat{T}_n \doteq -1.5 + (958 + 720\widehat{T}_r + 240\widehat{T}_r^2)^{1/2}.$$

Thus a substantial time saving results from using the nonparametric confidence interval based on sample size 360 rather than a corresponding confidence interval based on sample size 100 and all items failed. Only 62 items are failed if the nonparametric result is used. Also the amount of "information" obtained is at least as much as that for the corresponding "best" result based on a sample of 100 items all of which are failed. If testing 360 rather than 100 items does not unduly increase the cost for the experiment, use of the nonparametric result would seem desirable.

DERIVATIONS

This section contains an outline of the mathematical development of the asymptotic efficiency expressions used in computing the values for the case of a sample from $f(t; \xi, \lambda)$. The normality case need not be considered. The asymptotic efficiency values listed for the situation of normality and known variance were obtained directly from [8].

For the situations considered, the statistics encountered have distributions which are asymptotically normal. The asymptotic efficiency discussion presented in [8] shows that for this case it is sufficient to consider ratios of asymptotic variances in determining the asymptotic efficiencies of the nonparametric results. Also the material of [8] shows that the asymptotic variance of the statistic $t([nq])$, where $[nq]$ denotes the largest integer $\leqq nq$, can be used for the nonparametric results when θ_q is the percentage point of interest. That is, if θ_q is the percentage point of interest, the asymptotic efficiency of a nonparametric result (expressed as a fraction) equals the asymptotic variance of the statistic for the corresponding optimum result divided by the asymptotic variance of the order statistic $t([nq])$. Thus the problem of determining the asymptotic efficiencies of the nonparametric results for the situation of a sample from $f(t; \xi, \lambda)$ is reduced to that of obtaining the asymptotic variance of $t([nq])$, the asymptotic variance of the optimum statistic for the case where all the sample values are utilized, and the asymptotic variance of the optimum statistic for the case where only $t(1), t(2), \cdots, t([nq])$ are used.

First let us evaluate $\theta_q = \theta_q(\xi, \lambda)$ in terms of the unknown parameter ξ and the known parameter λ. To do this, consider the statistical population with probability density function

$$f(y; \lambda) = y^{\lambda-1}e^{-y}/\Gamma(\lambda), \qquad (1)$$

where $y \geqq 0$. Let $\phi_q(\lambda)$ be the $100q$ percentage point of this population. Then since

$$\int_0^{\xi\phi_q(\lambda)} \frac{t^{\lambda-1}e^{-t/\lambda}}{\Gamma(\lambda)}dt = \int_0^{\phi_q(\lambda)} \frac{y^{\lambda-1}e^{-y}}{\Gamma(\lambda)}dy = q,$$

the value of θ_q equals $\xi\phi_q(\lambda)$.

Now consider determination of the asymptotic variance of the order statistic $t([nq])$. Since n is the sample size, $t([nq])$ is the sample $100q$ percentage point. Thus the asymptotic variance of $t([nq])$ has the value

$$\frac{q(1-q)}{n\{f(\theta_q;\xi,\lambda)\}^2} = \frac{q(1-q)\{\Gamma(\lambda)\}^2\xi^2}{n\{\phi_q(\lambda)\}^{2(\lambda-1)}e^{-2\phi_q(\lambda)}}.$$

This is a direct application of the results of [1, p. 369].

Next consider the asymptotic variance of the optimum statistic for the case where all n sample values are used. Since $\theta_q=\xi\phi_q(\lambda)$, this asymptotic variance equals $\{\phi_q(\lambda)\}^2$ times the asymptotic variance of the optimum statistic for the unknown parameter ξ. On the basis of the material of [1, pp. 500–505], the asymptotic variance of the optimum statistic for ξ equals the reciprocal of

$$n\int_0^\infty\left(\frac{\partial \log_e f(t;\xi,\lambda)}{\partial\xi}\right)^2 f(t;\xi,\lambda)dt = \frac{n\lambda}{\xi^2}.$$

Thus the asymptotic variance of the optimum statistic for the case where all the sample values are used has the value $\xi^2\{\phi_q(\lambda)\}^2/n\lambda$. Consequently the asymptotic efficiency (expressed as a fraction) of the nonparametric results for this case is

$$\{\lambda(\phi_q(\lambda))^{2\lambda}e^{-2\phi_q(\lambda)}\}/\{q(1-q)(\Gamma(\lambda+1))^2\}.$$

The factor values listed in Table 2 for the situation of a sample from $f(t;\xi,\lambda)$ with λ known were computed from the reciprocal of this asymptotic efficiency expression.

Finally let us consider the asymptotic variance of the optimum statistic for the case where only the $[nq]$ smallest order statistics of the sample are used. This asymptotic variance equals $\{\phi_q(\lambda)\}^2$ times the asymptotic variance of the optimum statistic for ξ. Using the results of [4], the asymptotic variance of the optimum statistic for ξ equals the reciprocal of

$$n\int_0^{\phi_q}\left(\frac{\partial \log_e f(t;\xi,\lambda)}{\partial\xi}\right)^2 f(t;\xi,\lambda)dt + \frac{n}{1-q}\left(\int_0^{\phi_q}\frac{\partial f(t;\xi,\lambda)}{\partial\xi}dt\right)^2$$

$$= \frac{n\lambda^2}{\xi^2}\left[q - 2\int_0^{\phi_q(\lambda)}\frac{y^\lambda e^{-y}}{\Gamma(\lambda+1)}dy + (1+1/\lambda)\int_0^{\phi_q(\lambda)}\frac{y^{\lambda+1}e^{-y}}{\Gamma(\lambda+2)}dy\right.$$

$$\left. + \frac{1}{1-q}\left(q - \int_0^{\phi_q(\lambda)}\frac{y^\lambda e^{-y}}{\Gamma(\lambda+1)}dy\right)^2\right].$$

From this it follows that the asymptotic efficiency (expressed as a fraction) of the nonparametric results is

$$\{\phi_q(\lambda)\}^{2\lambda}\{\Gamma(\lambda+1)\}^{-2}e^{-2\phi_q(\lambda)}/q(1-q)$$

$$\cdot\left[q - 2\int_0^{\phi_q(\lambda)}\frac{y^\lambda e^{-y}}{\Gamma(\lambda+1)}dy + (1+1/\lambda)\int_0^{\phi_q(\lambda)}\frac{y^{\lambda+1}e^{-y}}{\Gamma(\lambda+2)}dy\right.$$

$$\left. + \frac{1}{1-q}\left(q - \int_0^{\phi_q(\lambda)}\frac{y^\lambda e^{-y}}{\Gamma(\lambda+1)}dy\right)^2\right]^{-1}$$

for the case where only $t(1), \cdots, t([nq])$ are utilized. This expression was used in computing the asymptotic efficiences which showed that the values for normality and known variance represent efficiency lower bounds for the situation of a sample from $f(t; \xi, \lambda)$, with λ known.

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ESTIMATES OF BOUNDED RELATIVE ERROR FOR THE RATIO OF VARIANCES OF NORMAL DISTRIBUTIONS

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1

STATISTICAL methods for comparing the variability of two competing industrial processes are often just as important as statistical methods for comparing location parameters. For example, a manufacturer of half-inch ball bearings is not likely to be interested in a proposed modification of his manufacturing process which promises to produce bearings of larger average diameter, though he may welcome a change which can be expected to produce bearings which are substantially less variable in size.

In this paper we assume that the characteristic whose variability is in question behaves like a random variable which is normally distributed with variance σ_1^2 for the first process and variance σ_2^2 for the second. We assume further that the σ^2s are unknown but that we can obtain appropriate data on the output of each process sufficient to enable us to make an unbiased estimate $\hat{\sigma}_1^2$ of σ_1^2 and an unbiased estimate $\hat{\sigma}_2^2$ of σ_2^2 such that $n_1\hat{\sigma}_1^2/\sigma_1^2$ and $n_2\hat{\sigma}_2^2/\sigma_2^2$ are independently distributed like chi-square with n_1 and n_2 degrees of freedom respectively. We then wish to estimate the ratio σ_1^2/σ_2^2.[1]

Under these assumptions it is common practice to use $\hat{\sigma}_1^2/\hat{\sigma}_2^2$ as the desired estimate of σ_1^2/σ_2^2, and to describe the precision of the estimate by using tables of the F distribution to determine numbers h_α and H_α such that for a preassigned confidence level α we have

$$P_r\left\{\frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} < h_\alpha \frac{\hat{\sigma}_1^2}{\sigma_2^2}\right\} = P\left\{\frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} > H_\alpha \frac{\sigma_1^2}{\sigma_2^2}\right\} = \frac{1-\alpha}{2}.$$

Experience has shown, however, that this solution to the problem is a rather awkward one from the standpoint of practical application, particularly when the estimate of σ_1^2/σ_2^2 is to be used as one of the terms in a chain of calculations. In this case the engineer will often prefer an estimate about which he can say, for example, "The chances are 19 in 20 that the ratio of this estimate to its true value will lie between 1.1 and 1/1.1".

In the terminology of Blackwell and Girshick an estimate with the last mentioned property is said to be of bounded relative error. It has been shown[2] that a minimax estimate, which is of bounded relative

* I am indebted to Joseph F. Daly of Stanford University and the Bureau of the Census for helpful comments. The research reported and the computations shown in this paper were done at the Statistical Laboratory, Stanford University.

[1] E. J. G. Pitman, "Tests of hypotheses concerning location and scale parameters", *Biometrika*, 31 (1939), 200–15, for a more general statement of this problem.

[2] D. Blackwell and M. A. Girshick, *Theory of Games and Statistical Decisions*. (New York: John Wiley and Sons, 1954), pp. 316–23.

error with a confidence coefficient which is independent of the parameter, always exists if one is dealing with a distribution in which the only unknown parameter is a scale parameter. What follows is a straightforward application of the general theory given in the present case. A similar application of the theory has been given by Girshick et al., where the problem of finding an optimum estimate of bounded relative error for the variance of a single normal distribution is discussed.[3] The advantages of estimates of bounded relative error in this latter case are pointed out,[4] without, however, the development of a theory for obtaining optimum estimates of this type.

2

Writing $\tau = \sigma_1^2/\sigma_2^2$, we shall consider estimates $\hat{\tau}$ of the form

$$\hat{\tau} = \beta \cdot \frac{\delta_1^2}{\delta_2^2} = \frac{1}{b}\,\frac{\delta_1^2}{\delta_2^2}\, b > 0$$

where we assume that $n_1\delta_1^2/\sigma_2^2$ and $n_2\delta_2^2/\sigma_2^2$ are independently distributed like chi-square with n_1 and n_2 degrees of freedom respectively. Under these assumptions it is well known that the quantity $b\hat{\tau}/\tau$ has the F distribution with n_1 and n_2 degrees of freedom. The condition that $\hat{\tau}$ be of bounded relative error with confidence α may be expressed by the relation

$$P\left\{\frac{1}{K} \leqq \frac{\tau}{\hat{\tau}} \leqq K\right\} = \alpha(K)$$

for $K>0$, where $\alpha(K)$ does not depend on τ. But this is the same as

$$P\left\{\frac{b}{K} \leqq \frac{b\hat{\tau}}{\tau} \leqq bK\right\} = \int_{b/K}^{bK} \phi(x, n_1, n_2)dx = \psi_K(b, n_1, n_2) \qquad (1.1)$$

where $\phi(x, n_1, n_2)$ is the F density with n_1 and n_2 degrees of freedom.

We observe that $\psi_K(b, n_1, n_2)$ does not depend on τ. Moreover, for fixed n_1, n_2 and K we obtain the best estimate $\hat{\tau}$ by choosing that value b^* of b which maximizes

$$\psi_K(b, n_1, n_2) = P\left\{1/K \leqq \frac{\hat{\tau}}{\tau} \leqq K\right\}.$$

Differentiating (1.1) with respect to b and putting the derivative equal to zero, we obtain the condition

$$K\phi(bK, n_1, n_2) = \frac{1}{K}\,\phi(b/K, n_1, n_2). \qquad (1.2)$$

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[3] M. A. Girshick, H. Rubin, and R. Sitgreaves, "Estimates of bounded relative error in particle counting", *Annals of Mathematical Statistics*, 26 (1955), 276–85.

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By making use of the explicit form

$$\phi(x, n_1, n_2) = \frac{1}{\beta\left(\frac{n_1}{2}, \frac{n_2}{2}\right)} \cdot \left(\frac{n_1}{n_1}\right)^{n_1/2} x^{(n_1/2)-1} \left(1 + \frac{n_1}{n_2} x\right)^{-(n_1+n_2)/2} \tag{1.3}$$

of the F density we then solve (1.2) for the maximizing value b^* of b, and find

$$b^* = \frac{K^{n_1/n_1+n_2} - K^{-n_1/n_1+n_2}}{K^{n_1/n_1+n_2} - K^{-n_2/n_1-n_2}} \cdot \frac{n_2}{n_1}. \tag{1.4}$$

Hence the optimum estimate $\hat{\tau}^*$ of σ_1^2/σ_2^2 is

$$\hat{\tau}^* = \frac{K^{1/1+\theta} - K^{-1/1+\theta}}{K^{\theta/1+\theta} - K^{-1/1+\theta}} \cdot \theta \cdot \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} = \beta^* \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} \tag{1.5}$$

where we have written θ for the ratio n_1/n_2. The associated confidence coefficient is

$$\psi_K(b^*, n_1, n_2) = P\left\{\frac{1}{K} \leqq \frac{\hat{\tau}^*}{\tau} \leqq K\right\} = \int_{b^*/K}^{b^*K} \phi(x, n_1, n_2) dx. \tag{1.6}$$

Several interesting points may now be noted. In the first place, the derivation of (1.5) and (1.6) makes no assumptions about $\hat{\sigma}_1$ and $\hat{\sigma}_2$ other than that $n_1\hat{\sigma}_1^2/\sigma_1^2$ and $n_2\hat{\sigma}_2^2/\sigma_2^2$ be independently distributed like chi-square with n_1 and n_2 degrees of freedom. Thus the result if equally applicable to cases in which the quantities σ_1^2 and σ_2^2 are, for example, conditional variances or variances about a regression function, and where the estimates $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ need not be the conventional mean square deviations from the sample averages. Secondly, it can be seen from (1.5) that, for fixed K, the optimal value of the factor β^* which multiples $\hat{\sigma}_1^2/\hat{\sigma}_2^2$ depends only on the ratio of the sample sizes and is unity when $n_1 = n_2$. Values of β^* for selected values of K and selected values of $\theta = n_1/n_2$ are given in Table 1. Values of the confidence coefficient $\alpha = \psi_K(b^*, n_1, n_2)$, which depends in a more complicated way on K, n_1 and n_2, may be obtained from the charts in the Appendix. Finally it follows from (1.1) and (1.3) that when $n_1 = n_2$ we have

$$P\left\{\frac{\hat{\tau}^*}{\tau} < \frac{1}{K}\right\} = P\left\{\frac{\hat{\tau}^*}{\tau} > K\right\} = \frac{1-\alpha}{2}$$

so that when the sample sizes are equal the classical estimating procedure using equal tails of the F distribution is equivalent to the procedure using $\hat{\tau}^*$.

3

There are at least two types of situations in which Table 1 and the charts in the appendix may prove useful. In the first situation the ex-

TABLE 1

VALUES OF β^* TO BE USED WITH THE ESTIMATE $\hat{\tau} = \beta^* \hat{\sigma}_1^2/\sigma_2^2$, FOR SELECTED VALUES OF $\theta = n_1/n_2$ AND FOR SELECTED VALUES OF THE RELATIVE ERROR BOUND K

K \ θ	1.0	1.5	2.0	5.0	10.0	100.0
1.01	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.02	1.0000	1.0000	1.0000	1.0000	.9999	.9999
1.05	1.0000	.9999	.9999	.9997	.9997	.9996
1.10	1.0000	.9997	.9995	.9990	.9988	.9985
1.25	1.0000	.9983	.9972	.9945	.9932	.9919
1.30	1.0000	.9977	.9962	.9924	.9907	.9888
1.40	1.0000	.9962	.9937	.9875	.9847	.9817
1.50	1.0000	.9946	.9909	.9820	.9779	.9736

periment has already been performed, and the statistician is asked to do the best he can with the data. In this case the statistician can enter Table 1 with the ratio $n_1/n_2 = \theta$, and for a suitable value of K determine β^* and thus obtain his best estimate $\hat{\tau}^*$. The confidence level associated with the resulting estimate can then be read from the chart in the appendix corresponding to the selected value of K by noting the position of the point (n_1, n_2) relative to the curves on the chart. In the second situation the statistician may be asked to determine, before any observations are made, appropriate values of n_1 and n_2 in such a way as to yield a prescribed level of accuracy at minimum cost. In this case the statistician should go first to the chart in the appendix corresponding to the selected value of K, and find the curve corresponding to the confidence level. If $C(n_1, n_2)$ denotes the cost of getting estimates $\hat{\sigma}_1$ and $\hat{\sigma}_2$ with n_1 and n_2 degrees of freedom respectively, he can select from the curve several pairs of values (n_1, n_2), and by substituting them in the function $C(n_1 n_2)$ determine the approximate minimum and the corresponding values of n_1 and n_2. Or he can plot several level curves $C(n_1, n_2) = \text{constant}$ to the same scale as the chart, and thereby determine a value c_0 for the constant such that the curve $(C(n_1, n_2) = c_0$ is tangent to the chosen curve in the appendix. The point of tangency will then yield the desired values of n_1 and n_2.

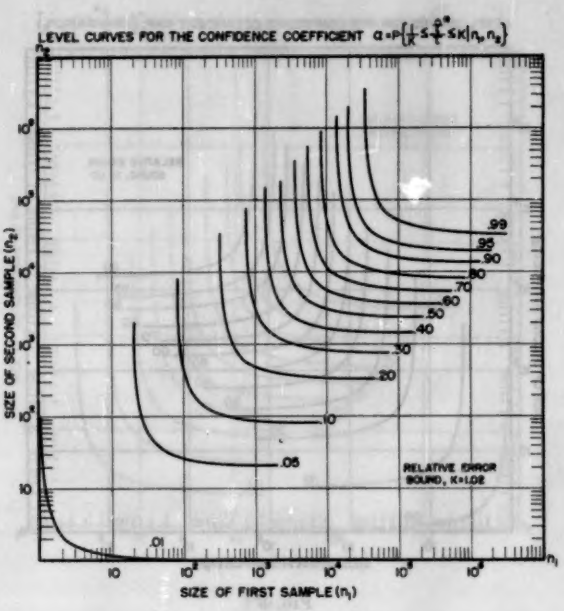
LEVEL CURVES FOR THE CONFIDENCE COEFFICIENT $\alpha = P\left\{\frac{1}{K} \le \frac{s_1^2}{s_2^2} \le K \mid n_1, n_2\right\}$
SIZE OF SECOND SAMPLE (n_2)
SIZE OF FIRST SAMPLE (n_1)
.99
.95
.90
.80
.70
.60
.50
.40
.30
.20
.10
.05
.01
RELATIVE ERROR BOUND, K=1.02

FIG. 1

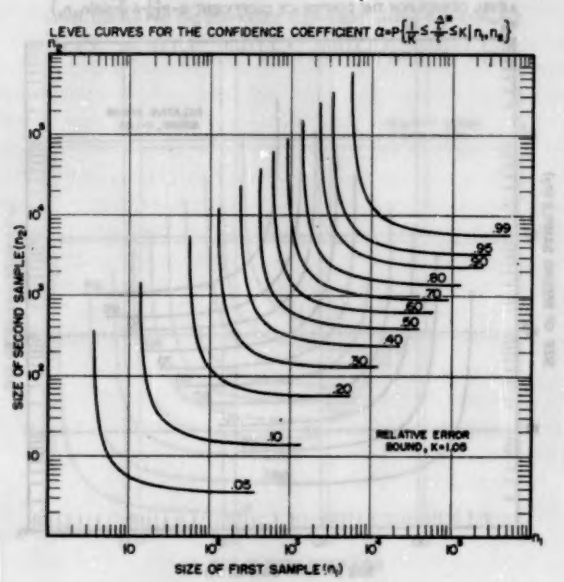
LEVEL CURVES FOR THE CONFIDENCE COEFFICIENT $\alpha = P\left\{\frac{1}{K} \le \frac{s_1^2}{s_2^2} \le K \mid n_1, n_2\right\}$
SIZE OF SECOND SAMPLE (n_2)
SIZE OF FIRST SAMPLE (n_1)
.99
.95
.90
.80
.70
.60
.50
.40
.30
.20
.10
.05
RELATIVE ERROR BOUND, K=1.05

FIG. 2

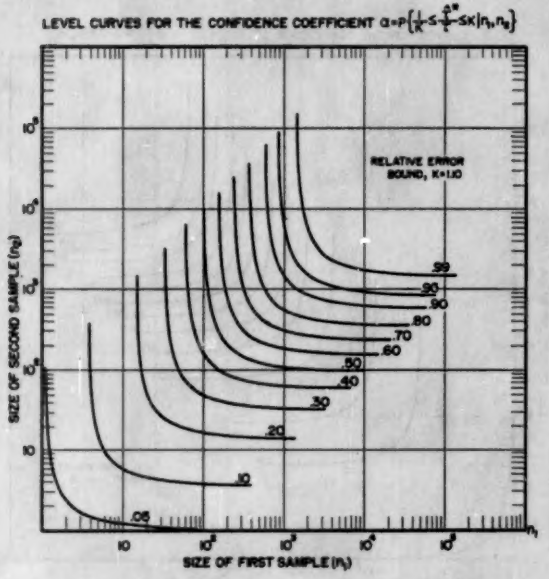
LEVEL CURVES FOR THE CONFIDENCE COEFFICIENT $\alpha = P\left\{\frac{1}{K} \le \frac{\hat{\sigma}^2}{\sigma^2} \le K \mid n_1, n_2\right\}$
RELATIVE ERROR BOUND, K=1.10
.99
.95
.90
.80
.70
.60
.50
.40
.30
.20
.10
.05
SIZE OF SECOND SAMPLE (n_2)
SIZE OF FIRST SAMPLE (n_1)

FIG. 3

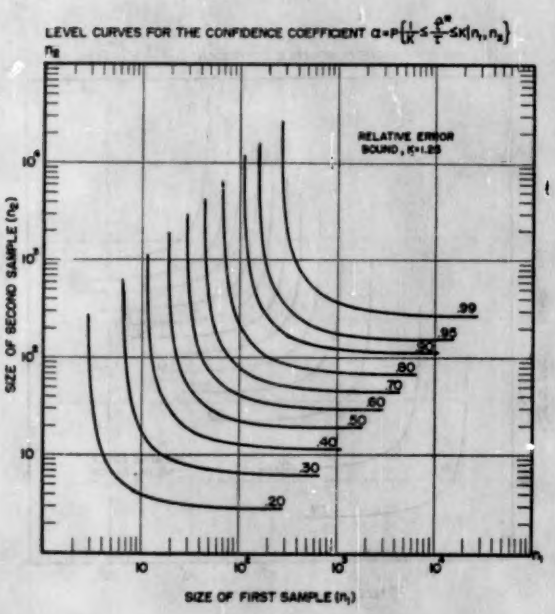
LEVEL CURVES FOR THE CONFIDENCE COEFFICIENT $\alpha = P\left\{\frac{1}{K} \le \frac{\hat{\sigma}^2}{\sigma^2} \le K \mid n_1, n_2\right\}$
RELATIVE ERROR BOUND, K=1.25
.99
.95
.90
.80
.70
.60
.50
.40
.30
.20
SIZE OF SECOND SAMPLE (n_2)
SIZE OF FIRST SAMPLE (n_1)

FIG. 4

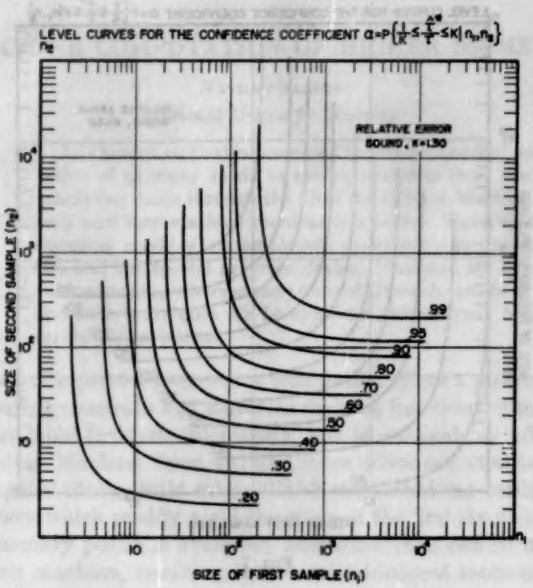
LEVEL CURVES FOR THE CONFIDENCE COEFFICIENT
RELATIVE ERROR BOUND, K=1.30
.99
.95
.90
.80
.70
.60
.50
.40
.30
.20
SIZE OF SECOND SAMPLE
SIZE OF FIRST SAMPLE

FIG. 5

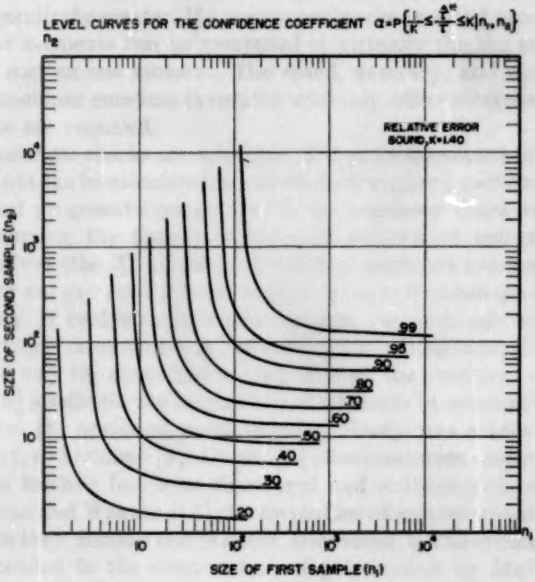
LEVEL CURVES FOR THE CONFIDENCE COEFFICIENT
RELATIVE ERROR BOUND, K=1.40
.99
.95
.90
.80
.70
.60
.50
.40
.30
.20
SIZE OF SECOND SAMPLE
SIZE OF FIRST SAMPLE

FIG. 6

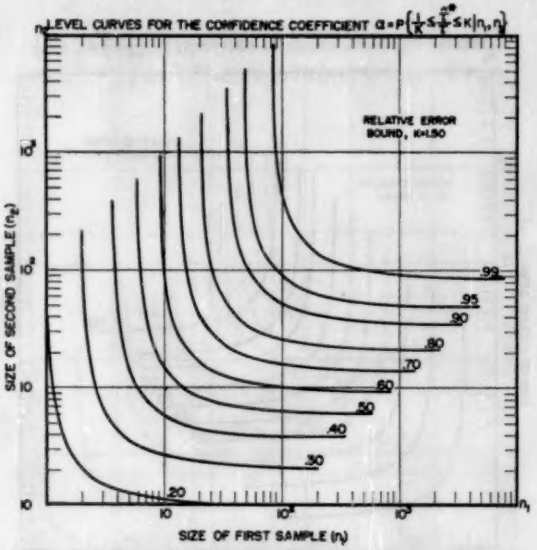
LEVEL CURVES FOR THE CONFIDENCE COEFFICIENT $\alpha = P\left(\frac{1}{K} \le \frac{\bar{x}^*}{\bar{x}} \le K \mid n_1, n_2\right)$
RELATIVE ERROR BOUND, K=1.50
.99
.95
.90
.80
.70
.60
.50
.40
.30
.20
SIZE OF SECOND SAMPLE (n_2)
SIZE OF FIRST SAMPLE (n_1)
10^3
10^2
10
1
n_2
n_1

FIG. 7

MACHINE COMPUTATION OF HIGHER MOMENTS

NATHAN JASPEN
National League for Nursing

An efficient method is presented for the economical and rapid computation of as many as six to twelve moments from a single run of the tabulating cards through the IBM Accounting Machine Type 402. The only auxiliary machine required is a sorter. Three features of the accounting machine are employed: progressive summation, total transfer, and the special program device. Moments are available from the successive progressive summations of either the scores or the card count, and both techniques can be applied simultaneously to provide a check on the computations.

THE method of repeated progressive summation offers a powerful technique for computing moments and statistics that are functions of moments. This method is available for desk calculators, but is particularly adapted to the IBM Accounting Machine Type 402 and more advanced models.

It is possible to obtain, on a single run of the tabulating cards through the machine, factors which readily yield the sums of the first six to twelve powers of X. If a summary punch is available, additional runs can be made through the accounting machine, resulting in several additional moments from each run, up to the full counter capacity of the machine. The maximum capacity of the IBM Accounting Machine Type 407 is considerably greater than that of the Type 402, and the number of moments that can be computed from one run is correspondingly greater. If a permanently wired control panel is available, several higher moments can be computed in virtually the the same time and at the same cost as one moment. The speed, accuracy, and economy of the accounting machines compare favorably with any other equipment when several moments are required.

Several numerical checks are available. The most important of these is that all the moments can be calculated in two distinct ways: by performing the process of repeated progressive summation on the frequency (card count) column, and by performing the process of repeated progressive summation on the scores themselves (the Xf column). If only six moments are required, and if the factors are not excessively large, both of these techniques can be performed simultaneously. If twelve moments are desired, the cards can be run through the machine twice, once with only the card count feeding through the counters, and once with only the score field feeding through the counters.

Elderton [5] attributes the calculation of moments by summation to Hardy, whose account [8] appeared posthumously. Hardy was evidently indebted, at least in part, to Lidstone [9]. Dwyer [2] cites some even earlier sources; and no doubt the method has been discovered and rediscovered by many individuals. Rozran and Wagner [13] cite several investigators who discovered the method before they, Rozran and Wagner, discovered it. The summation method has been extended to the computation of polynomials by McPherson [10]. Toops [14] brought the progressive summation method to the attention of psychologists and educators, and has made other contributions in this area

[15, 16, 17, 18]. Related contributions have been made by Rozran and Wagner [13], Dwyer and Meacham [4], Dwyer [3], and Frankel [7].

Mendenhall and Warren [11, 12] were the first to publish a tabulating machine method for computing the moments required in correlation by progressive summation. Brandt [1] also learned of a progressive summation method as a punched-card application; his sources were employees of International Business Machines Corporation, but it is not clear where they learned it. Warren [19] applied the method of progressive summation, using tabulating equipment, to the computation of least square lines.

Ellis and Riopelle [6] developed a punched-card method for computing higher moments. Their method involved successive summary punching, collating a variable number of blank cards behind these summary cards, gang-punching the data from the summary cards into the blank cards, and totalling the entries on these summary cards.

The advantage of the method to be described in this paper is that several moments can be computed from a single tabulating run, without the use of blank cards, the collator, or the summary punch, and with no more demands on the operator than are occasioned by a single progressive summation for $\sum X^2$. This method could not have been adapted to the tabulating equipment which was available to earlier workers.

Moments are useful parameters for describing frequency distributions. Occasionally it is desirable to obtain moments of high order. For instance, the standard error of the second moment involves the fourth moment; the standard error of the third moment, which is used in measures of skewness, involves the sixth moment; and the standard error of the fourth moment, used in measures of kurtosis, involves the eighth moment.

NUMERICAL ILLUSTRATION. Table 1 is a tabulation resulting from the running of a small deck of IBM cards through the tabulator. This deck consists of eleven scores: 2 five's, 4 three's, 3 two's, and 2 one's.

Column a in Table 1 represents the progressive summation of the frequency column. Column b represents the progressive summation of column a. Similarly for columns c, d, e, f.

Column A in Table 1 represents the progressive summation of the Xf column. Column B represents the progressive summation of column A. Similarly for columns C, D, E, F.

Each cell may be identified by its row and column. For instance, N equals a_1. $\sum X$ equals $b_1 = A_1$. To simplify notation, if no subscript is specified, it will be understood to be 1. Thus, $e = 217$, $F = 1550$.

If there are some zero scores, a row corresponding to $X = 0$ will also print. In that case, N would equal a_0. But X would equal $b_1 = A_1$, as before.

Rationale of the procedure of repeated progressive summation. Consider a single score p. The method of repeated progressive summation applied to the test score results in the matrix shown in Table 2. It will be noted that the coefficients are those which occur in the binomial expansion and in the diagonals of Pascal's triangle.

The general terms are most readily obtained by the methods of the calculus of finite differences. The terms in the bottom row are all expressions in p of

TABLE 1

ILLUSTRATION OF SIX PROGRAMS OF PROGRESSIVE SUMMATION

Score	Card Count Progressive Summations						Score	Test Score Progressive Summations					
	First	Second	Third	Fourth	Fifth	Sixth		First	Second	Third	Fourth	Fifth	Sixth
X	a	b	c	d	e	f	X	A	B	C	D	E	F
5	2	2	2	2	2	2	5	10	10	10	10	10	10
4	2	4	6	8	10	12	4	10	20	30	40	50	60
3	6	10	16	24	34	46	3	22	42	72	112	162	222
2	9	19	35	59	93	139	2	28	70	142	254	416	638
1	11	30	65	124	217	356	1	30	100	242	496	912	1550

TABLE 2

PROGRESSIVE SUMMATIONS OF Xf FOR A SINGLE SCORE p

X	Xf	A	B	C	D
p	p	p	p	p	p
$p-1$	0	p	$2p$	$3p$	$4p$
$p-2$	0	p	$3p$	$6p$	$10p$
$p-3$	0	p	$4p$	$10p$	$20p$
$\vdots$					
$p-n+1$	0	p	np	$\frac{n^2+n}{2}p$	$\frac{n^3+3n^2+2n}{6}p$
$\vdots$					
1	0	p	p^2	$\frac{p^2+p^2}{2}$	$\frac{p^4+3p^3+2p^2}{6}$

degree one greater than the general term. These powers of p can be expressed in terms of the entries in the bottom row of cells, corresponding to $X=1$.

$$\begin{aligned} p^2 &= B \\ p^3 &= 2C - B \\ p^4 &= 6D - 6C + B. \end{aligned} \tag{1}$$

Although the matrix in Table 2 is symmetric, the sum of several such matrices for different values of p is not symmetric, since the several matrices are superimposed with the A_1 cell rather than the A_p cell as the pivot. When N single score matrices are summed, the terms in row 1 become linear functions of sums of powers.

A matrix similar to Table 2 in every respect is produced if the frequency or card count (1) rather than the score (p) is progressively summed, except that every term in the score matrix is p times the corresponding term in the card count matrix. This is shown in Table 3.

From this table also we can determine p, p^2, and p^3.

Determination of the moments about the origin. The following set of equations provides the moments about the origin, μ_1', μ_2', etc.:

TABLE 3

PROGRESSIVE SUMMATIONS OF f FOR A SINGLE SCORE p

X	f	a	b	c	d
p	1	1	1	1	1
$p-1$	0	1	2	3	4
$p-2$	0	1	3	6	10
$p-3$	0	1	4	10	20
$\vdots$					
$p-n+1$	0	1	n	$\frac{n^2+n}{2}$	$\frac{n^3+3n^2+2n}{6}$
$\vdots$					
1	0	1	p	$\frac{p^2+p}{2}$	$\frac{p^3+3p^2+2p}{6}$

$$
\begin{aligned}
N &= a \\
N\mu_1' &= \sum X = b \\
N\mu_2' &= \sum X^2 = -b + 2c \\
N\mu_3' &= \sum X^3 = b - 6c + 6d \\
N\mu_4' &= \sum X^4 = -b + 14c - 36d + 24e \\
N\mu_5' &= \sum X^5 = b - 30c + 150d - 240e + 120f.
\end{aligned} \tag{2}
$$

Similarly

$$
\begin{aligned}
\sum X &= A \\
\sum X^2 &= B \\
\sum X^3 &= -B + 2C \\
\sum X^4 &= B - 6C + 6D \\
\sum X^5 &= -B + 14C - 36D + 24E \\
\sum X^6 &= B - 30C + 150D - 240E + 120F.
\end{aligned} \tag{3}
$$

It will be observed that the coefficients alternate in sign, and that the algebraic sum of the coefficients is always 1. Disregarding sign, the coefficient of B (and b) is always 1. Disregarding sign, the coefficient of each C is twice the sum of the coefficients of B and C in the previous equation. Disregarding signs, the coefficient of each D is three times the sum of the coefficients of C and D in the previous equation. Disregarding signs, the coefficient of E is four times the sum of the coefficients of D and E in the previous equation. The end terms are consequently factorials.

The moments can also be calculated from any of the diagonals, proceeding from the lower left to upper right, and beginning with a term in row 1. In particular, if we begin with b_1 or B_1, we find that

$$
\begin{aligned}
\sum X &= b_1 \\
\sum X^2 &= b_1 + 2c_2 \\
\sum X^3 &= b_1 + 6c_2 + 6d_3 \\
\sum X^4 &= b_1 + 14c_2 + 36d_3 + 24e_4 \\
\sum X^5 &= b_1 + 30c_2 + 150d_3 + 240e_4 + 120f_5
\end{aligned} \tag{4}
$$

$$\begin{aligned}
\sum X^2 &= B_1 \\
\sum X^3 &= B_1 + 2C_2 \\
\sum X^4 &= B_1 + 6C_2 + 6D_3 \\
\sum X^5 &= B_1 + 14C_2 + 36D_3 + 24E_4 \\
\sum X^6 &= B_1 + 30C_2 + 150D_3 + 240E_4 + 120F_5.
\end{aligned} \tag{5}$$

Except for signs, the equations in (4) and (5) are similar to the equations in (2) and (3). Equations (4) and (5) are easier to use, because no negative terms are involved, because the numbers are smaller, and because fewer summations are necessary. A property of the diagonal equations is this: above the pth (top) row, all values are equal to zero. For instance, in Table 1, the values G_6, g_6, H_7, h_7, and so on, are all zero. No further tabulations are required, therefore, for the distribution in Table 1; the tabulation as shown is sufficient to compute moments of any higher order whatever. This reflects the fact that, since there are only five independent frequencies, five parameters are sufficient to specify completely the entire distribution, including all its moments.

The equations in (4) can be derived from the equations in (2), using the following information:

$$\begin{aligned}
c_1 &= b_1 + c_2 \\
d_1 &= b_1 + 2c_2 + d_3 \\
e_1 &= b_1 + 3c_2 + 3d_3 + e_4 \\
f_1 &= b_1 + 4c_2 + 6d_3 + 4e_4 + f_5.
\end{aligned} \tag{6}$$

These equations are also true if upper case letters are substituted for the lower case letters.

Computation of the moments about the mean. Usually moments about the origin are computed only for the purpose of determining the moments about the mean.

Moments about the mean can be computed directly from the following formulas:

$$\begin{aligned}
\mu_1 &= \frac{\sum x}{N} = 0 \\
\mu_2 &= \frac{\sum x^2}{N} = \frac{1}{N^2}\left[-Nb + 2Nc - b^2\right] = \sigma^2 \\
\mu_3 &= \frac{\sum x^3}{N} = \frac{1}{N^3}\left[N^2b - 6N^2c + 6N^2d + 3Nb^2 - 6Nbc + 2b^3\right] \\
\mu_4 &= \frac{\sum x^4}{N} = \frac{1}{N^4}\left[-N^3b + 14N^3c - 36N^3d + 24N^3e - 4N^2b^2\right. \\
&\qquad \left. + 24N^2bc - 24N^2bd - 6Nb^3 + 12Nb^2c - 3b^4\right].
\end{aligned} \tag{7}$$

If L is the lowest score, the formulas above are equally true in terms of b_L, c_L, etc. Therefore the card count matrix need not be carried below the lowest score.

Also the following:

$$\mu_2 = \frac{1}{N^2}\left[NB - A^2\right] = \sigma^2$$

$$\mu_3 = \frac{1}{N^3}\left[-N^2B + 2N^2C - 3NAB + 2A^3\right] \tag{8}$$

$$\mu_4 = \frac{1}{N^4}\left[N^3B - 6N^2C + 6N^2D + 4N^2AB - 8N^2AC + 6NA^2B - 3A^4\right].$$

Computation of the factorial moments. Factorial moments can also be determined from the progressive summation matrices. The expression $X^{[p]}$ is defined as $X(X-1)(X-2\cdots(X-p+1)$. The factorial moment $\mu_{(r)}'$ is equal to $1/N\sum X^{[r]}$. Factorial moments are useful in interpolation theory and curve fitting.

The factorial moments may be found from the following formulas:

$$\begin{aligned} N\mu'_{[1]} &= b_1 = A_1 \\ N\mu'_{[2]} &= 2c_2 = B_2 \\ N\mu'_{[3]} &= 6d_3 = 2C_3 \\ N\mu'_{[4]} &= 24e_4 = 6D_4 \\ N\mu'_{[5]} &= 120f_5 = 24E_5 \end{aligned} \tag{9}$$

and so on.

Computation of bivariate moments. The methods herein outlined are readily extended to include the computation of $\sum XY$, $\sum X^2Y$, $\sum X^3Y$, etc. It is only necessary to sort on X, and read Y into counter $6A$. Cross products in which the exponents of both X and Y exceed 1, such as $\sum X^2Y^2$, cannot be computed on a single run of the cards through the tabulator by the methods outlined in this paper.

Wiring diagram. The wiring diagram that produced Table 1, except for columns *e, f, E, F,* is shown in Figure 1. For still higher moments, the wiring pattern is continued. Three features of the machine are employed: progressive summation, total transfer, and the special program device. The special program device is an optional feature of the machine, which makes it possible to initiate any number of program cycles for any one program start. Without this feature, only three moments can be calculated on one run.

Details of the wiring are as follows:

1. The score field is wired to print and to counter entry 6A. Counter exit 6A is wired to print and to counter entry 6B. Counter exit 6B is wired to print and to counter entry 4A. Counter exit 4A is wired to print and to counter entry 4B. Counter exit 4B is wired to print.

2. The card count is wired to print and to counter entry 6C. Counter exit 6C is wired to print and to counter entry 6D. Counter exit 6D is wired to print and to counter entry 4C. Counter exit 4C is wired to print and to counter entry 4D. Counter entry 4D is wired to print.

3. Counters 6A and 6C add NX 80 cards only. The X 80 cards are a sequence deck, introduced to provide a control break on every possible score from the highest down to 1. This deck would not be necessary if there were no zero frequencies.

4. Counters 6A and 6C counter exits are suppressed on the read cycle.

5. The special program is wired, and the board is wired to provide for eleven program cycles.

6. The minor program start is impulsed from the comparing relay exit on each control break. A control break occurs when a score card of a new denomination passes the reading brushes. The intermediate program start is impulsed from the last card program start. The purpose of the intermediate program is to clear the machine after the last card has been read.

7. On program 1, counters 6A and 6C are impulsed to add progressively (to read out but not clear). The sums are transferred to counters 6B and 6D respectively. The counter exits of the second program counters 6B and 6D are suppressed, to prevent the printing of the first program totals from the second program counters. On programs 2 to 4, each successive pair of counters is controlled in the same way.

8. Spacing is suppressed for the first four programs, so that the four program steps for each control break print on the same line.

9. On program 5, the minor program is ended.

10. On programs 7 to 10, the four pairs of counters are each cleared in turn.

11. The printing of these clearances is suppressed. In this problem a pilot selector is picked up on the program 6 and the non-print hub is activated for all programs thereafter.

12. On program 11, the intermediate program is stopped.

For a larger number of moments than that shown in Figure 1, it is necessary to expand the number of program exit hubs. This can be done by picking up as many co-selectors as desired by a set of program hubs, then wiring five "all cycle" impulses into the common of the first co-selector, and from the normal of the co-selector to the common of the next co-selector, and so on. The transferred hubs of the several co-selectors then become additional program 1 hubs, program 2 hubs, and so on.

It is possible to wire the control panel so that the sum of each column appears below it. Each such sum should agree with the last entry of the column following. A separate counter would be required for the independent computation of each column sum. If these sums are desired, the second bank of counters can be used to provide them instead of the cumulations of f. The cumulations of f can be obtained on a second run, by wiring the card count into counter 6A in place of the score field.

An additional cycle of operations is possible after the last card has been read and before the counters are cleared, in order to solve equations of the type (1). In particular, it is desirable to perform the calculation $2c-b=\sum X^2$. This provides a check on B, which is also equal to $\sum X^2$. Both calculations for $\sum X^2$ can be made on the same tabulating run. Figure 2 shows the wiring for $\sum X^2$, based on the cumulations of the card count.

Features of Figure 2 are as follows:

1. The card count feeds into counter entry 6C. Counter exit 6C is wired through a co-selector into counter entry 6D, on program 1 only. Counter exit 6D is wired into counter entry 4C on program 2 only.

2. Co-selector 2 is picked up on program 1. Co-selector 3 is picked up on program 2.

3. If there are any zero scores, it is necessary to suppress total transfer from 6C to 6D and from 6D to 4C except for scores which are not equal to zero. Zero scores brought from the third reading brushes to the comparing relays fail to cause an unequal impulse. Unequal impulses, caused by non-zero scores, pick up pilot selector 9, and the transfer impulses for the first two programs are taken through this selector.

4. Co-selectors 4 and 7 are picked up on two programs after the end of the minor program and before the counters are cleared.

5. Counter 4C is impulsed to read out progressively (read out but not clear) on these two programs.

6. On these two programs only, the quantity in counter 4C is available for entry in counter 6D.

7. Counter 4C is impulsed to transfer into the minus control hubs of counter 6D.

8. Counter 6D is wired for subtraction.

When the counters are cleared, the quantity read out of counter 6D is $b-2c$. This is read out as a positive number.

This method is also available, with modifications in programming, if only three moments are desired. Alternatively, the digit emitter can be used to emit a 2 into one counter on every normal card cycle, and a 6 into another counter. By a system of programmed counter total transfers, including a set of transfers after the end of the minor program, the actual summations of sums of powers can be built up, instead of the components that must subsequently be entered into equations such as (1). This operation exhausts counter capacity very rapidly, however, and is scarcely worthwhile unless machines with much larger counter capacity become available.

Operating requirements. For progressive summation methods, it is customary and desirable to sort the cards from high to low, rather than from low to high. If the cards are sorted low to high, quantities B', C', etc., are obtained, instead of B, C, etc. If H is the highest score, then $\sum X^2=(H+1)\sum X-B'$.

A card should be present for every number between the highest score and 1. This is usually assured by sorting in a "dummy" sequence deck consisting of H cards, from H, the highest score, down to 1. The sequence deck must be identified in some way, such as an X punch in column 80. It is essential that no card in the test score deck have an X punch in column 80 if this is the column used to identify the sequence deck.

If, because of operator error or machine error, the number of control breaks is less than or greater than the full sequence of numbers from H to 1, the error will be detected by comparing the results from the two matrices.

Usually the sequence numbers in the sequence deck are punched in the same field as the scores in the test score deck, but it is possible to use a sequence

deck with the sequence numbers punched in a different field. In that case, wiring of the score and sequence fields from the reading brushes to the counters, type bars, and comparing relays must take place through the selectors. The selector for the third reading brushes to the comparing relays should be picked up by the X punch in column 80 routed from the second reading hubs to the X pick-up. The selector for the second reading brushes to the comparing relays should be picked up by the X punch in column 80 routed from the second reading hubs directly to the immediate pick-up. The two decks are each separately sorted from high to low, and the X cards are merged ahead of the NX cards. This technique is available if it is desired to maintain only one master deck of sequence cards for use with any deck of test score cards regardless of the field used for punching the test scores.

If the lowest score is greater than 1, it is desirable to insert "dummy" sequence cards down to 1. Alternatively, the scores can be coded; or moments can be computed about some point other than the origin; or modified versions of the formulas presented in this paper can be applied.

If some scores are negative, the scores may be coded by adding a constant so that no negative scores result. This is in effect done by the card count matrix if the rows are carried to a point above or below 1.

If there are no negative scores, but some zero scores do exist, it is important to remember that the only entry in the zero row of which cognizance should be taken is a, which is equal to N. $\sum X$ is equal to A_1 and b_1, not A_0 nor b_0.

An example of counter capacity. The following constants result from the progressive summations of a normal distribution of 10,000 two digit scores (the example is hypothetical but internally consistent):

$a =$	10,000
$b =$	500,000
$c =$	13,250,000
$d =$	246,500,000
$e =$	3,603,708,000
$A =$	500,000
$B =$	26,000,000
$C =$	713,000,000
$D =$	13,675,332,000

The 80 counter IBM tabulator Type 402 can handle this problem, with a few counters to spare, in a single run. The running time through the tabulator would be approximately an hour and a quarter. The constants are sufficient to calculate the first four moments.

Application of the progressive digiting method to more advanced machines. On more advanced IBM machines, such as the Accounting Machine Type 407 and the Magnetic Drum Data Processing Machine Type 650, it is not necessary to insert the dummy sequence X-80 deck, and solution of equations of

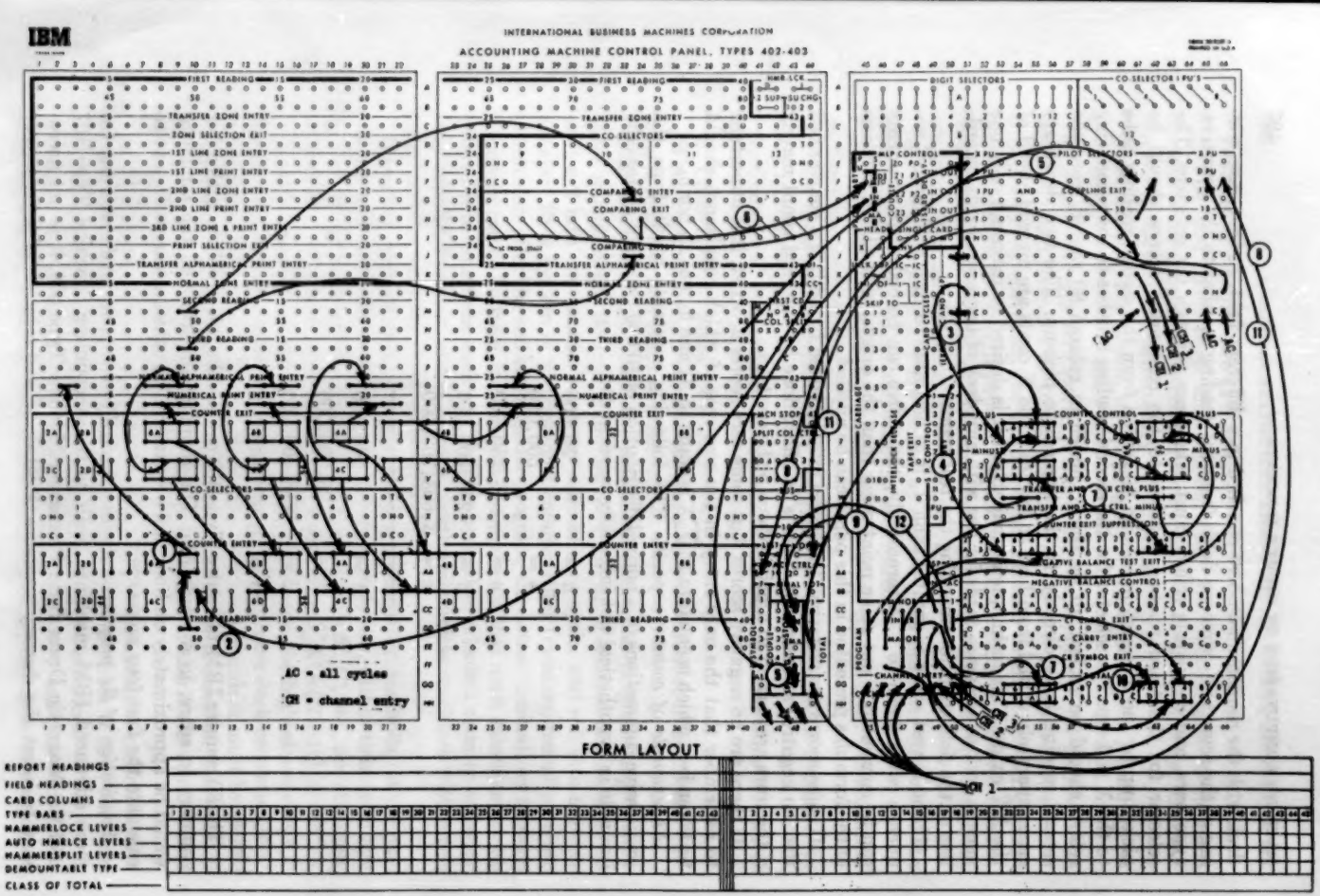

FIG. 1. Repeated Progressive Summations on the IBM Accounting Machine Type 402.

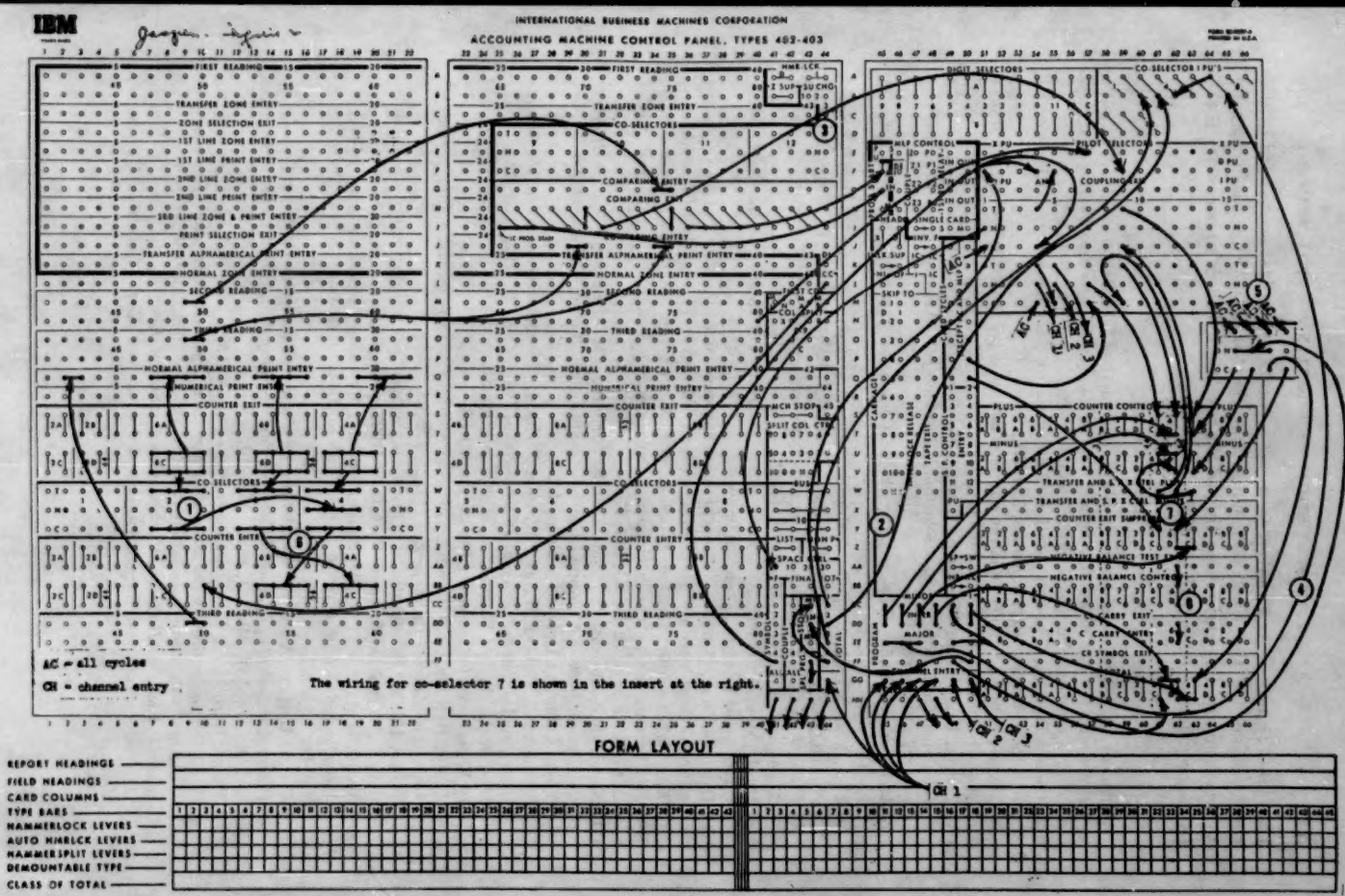

FIG. 2. Computation of $\sum X^3$ by Card Count Cumulations on the IBM Accounting Machine Type 402

the type (2) and (3) is a matter of a few instants as the last score card leaves the machine. On the Type 650, equations (4), (5), (7), (8), and (9) can be solved very rapidly; and pre-sorting of the test score deck is unnecessary if the test scores do not exceed three digits.

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SUMMARIES OF PAPERS DELIVERED AT THE 115TH ANNUAL MEETING OF THE AMERICAN STATISTICAL ASSOCIATION IN NEW YORK CITY, DECEMBER 27–29, 1955.

Edited by ARMEN A. ALCHIAN, *University of California, Los Angeles*

The present section contains all available abstracts of papers presented at the 1955 national meeting of the American Statistical Association in New York. The sequence of presentation here is alphabetical according to author.

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SUMMARIES OF PAPERS

The Urban Sex Ratio. CHESTER ALEXANDER, *Westminster College.*

During the 20th century the trend of the sex ratio of males to females in the large American cities has been steadily downward, but falling at an increasing slope, until at the present time urban females outnumber males by about 2,000,000. Nevertheless, the rapid rise of the number of females in cities should be viewed also from the perspective of the past. For 100 of the largest American cities the ratio has been over 100 until 1920, but since then it has fallen to 93.7. Another approach is to note that in 1900 37% of our largest cities had a ratio exceeding 100, but by 1950 only 4 cities had maintained a masculine numerical supremacy. At the tilt of the present slope, by 2000 A.D. this advantage will have disappeared, and even more than half of our cities will have 10% more women than men. Further investigation shows that the age of cities bears only a −.30 correlation to the sex ratios. Likewise, the small cities have no statistical distinctions worthy of note in their ratios. The recent large movement of women into the professions nearly all of which has taken place since 1920, does show a high relationship to the ratio. Multiple correlation by race groups yields −.34 as an index of racial effects of the ratio. These statistical facts express little of the ultimate effects on the changing ratio or the possible modifications in the social structure of our big cities.

Role of Local Census Tract Committees in Census Bureau Programs. C. E. BATSCHELET, *Bureau of the Census.*

During the year 1955, the Bureau of the Census assumed responsibility for all activities in connection with the Census Tract Program except the annual Census Tract Conference and the promotion of the use of tracts by local and national organizations. The Committee on Census Enumeration Areas of the American Statistical Association continues to direct these two activities.

It is the goal of the Bureau of the Census to have census tracts established in every city of 50,000 or more population and the surrounding metropolitan area before the next decennial census is taken. Of the 172 standard metropolitan areas in continental United States, 96 have now been tracted either in whole or in part, and tracting is in progress in many of the untracted areas.

The census tract key persons and the local census tract committees have been of great assistance to the Bureau of the Census in defining central business districts to be used in presenting statistics from the business census. It is hoped that a future program of mutual cooperation can be worked out which will prove to be of benefit to both the local tract committees and the Bureau of the Census.

National Monthly Help Wanted Advertising Index. WILLIAM A. BERRIDGE, *Metropolitan Life Insurance Company.*

This was an early sequel to the pilot project on factory labor turnover measurement, carried on by the Metropolitan Life Insurance Company and transferred in 1929 to the U. S. Bureau of Labor Statistics. The national sample index of help wanted ads was first published in 1930. Charted evidence shows how closely help wanted advertising still parallels the voluntary quit rate in factories, as measured by the B.L.S. index in its greatly expanded and improved form. Not only in World War II and major booms, but also in the strong seasonal movements and even in minor economic oscillations, the two indexes move closely in step nearly all of the time back to 1929, and before. The seasonally adjusted curve for help wanted ads shows, with great clarity, how faithfully the index continues to reflect the ups and downs of conditions as exemplified for instance by the latest recovery from the recession low of 1954 and continuing on upward to date. But jobs wanted proved to be a greatly inferior index of conditions, whether seasonal or cyclical. The present index is based upon about 60 newspapers, which are so distributed among 40 cities as to yield a reasonably balanced, representative sample of the national economy as a whole. The index is still of the link-chain type, wherein the several papers' links (or ratios of each month to its predecessor) are combined by use of the unweighted median. The chain relative is then referred to a base—the 1935–39 average for the longer range comparisons. The small sample reporting constitutes about 35% of the circulation of all newspapers in the U. S. The inner structure of the method, and the empirical behavior of the index results—especially in comparison

with economically related variables—warrant resting considerable faith in the validity of help wanted advertising as a measure of both cyclical and seasonal ebb and flow in labor markets.

Analysis of 2^n Factorial Experiments. ALLAN BIRNBAUM, *Columbia University.*

The standard methods of analysis of factorial experiments, especially 2^n experiments without replication, based on t and F statistics, are reviewed. Three categories of factorial experiments are distinguished, "measurement experiments," "weighing experiments," and "exploratory multifactor experiments." The usefulness and limitations of the standard methods are discussed for each category of experiment. An alternative method of analysis related to a special assumption of "hidden replication" is described.

An approximate graphical form of this method, described in Cuthbert Daniel's "Efficient Design of Experiments in Industry," to appear in *Proceedings of the Third Berkeley Symposium on Probability and Statistics*, is discussed and compared with the standard methods.

The Industrial Mobility of Labor as a Probability Process. ISADORE BLUMEN, MARVIN KOGAN, and PHILIP J. MCCARTHY *Cornell University.*

Probability models are discussed which are of general usefulness for the treatment of socio-economic processes such as industrial mobility, opinion change, and change in social status and these are compared with data on the inter-industry mobility of labor. The basic theoretical framework involves a combination of Markov processes and the probability approaches developed for accident statistics, absenteeism, and labor turnover. Particular attention is paid to the relation between short- and long-term movement patterns, the implications of short-term movement patterns for the eventual location of individuals, the treatment of differences in the amount of mobility, and the problem of attachment to a particular location or classification. The simplest model—a Markov chain—illustrates the fundamental ideas of a probability process and serves as a norm for the other work. This model makes no allowance for the development of industrial attachment. A modified model which does introduce attachment in a simple way is then treated and compared with our data. Further possible forms of modifications are discussed. A large subsample of records of a sample of workers obtained from the Continuous Work History Sample of the Bureau of Old-Age and Survivors Insurance form the empirical basis of the treatment.

Recent Studies of Adequacy of Benefits. PHILIP BOOTH. *Department of Labor.*

A pilot study of a sample of 400 unemployment insurace beneficiaries conducted in Pittsburgh, Pa., in 1954 disclosed that, while unemployed, these workers spent substantially more for food, shelter, and other items in the family budget than the received in unemployment insurance. Almost all claimants in the sample reduced expenditures after loss of work. The survey showed that weekly benefits represented less than 60 per cent of these reduced expenditures for most single claimants and less than 45 per cent for most 4-person families with their chief wage earner out of work. Claimant families supplemented their benefit checks by drawing upon other personal resources or by borrowing. This included using savings, cashing government bonds, cashing in or borrowing on insurance policies, borrowing from banks or using other sources of credit. It was emphasized that because of the experimental nature and limited scope of the survey, the findings should be used with caution so far as regional and national application is concerned. It should also be noted that the survey was made before the recent amendments of the Pennsylvania law increasing the maximum weekly benefit from $30 to $35.

Analysis of the results showed:

1. Before job loss, average weekly earnings of claimants before taxes amounted to $63 for single individuals and $76 for main wage earners in 4-person families. For all claimants, the average was $71.

2. Singe wage earners and chief wage earners reported that income dropped about 65 per cent after the onset of unemployment. These claimant groups reported a substantial cut in expenditures, but not so much as the fall in income. Expenditures of single claimants declined 32 per cent and those of chief wage earner claimants about 28 per cent.

3. The unemployment insurace checks represented all or nearly all of the income available to most single and main wage earner claimants.

4. About 60 per cent of the single claimants and 90 per cent of the chief wage earners received the maximum benefit of $30. For all claimants in the sample, the average weekly benefit was $28.

5. About 85 per cent of the claimants received weekly checks amounting to less than 50 per cent of their average weekly wages. The survey showed that 7 per cent of the chief wage earners and 37 per cent of the single claimants did receive payments equalling at least half their gross average weekly wages. An additional 50 per cent of the chief wage earners and 40 per cent of the single claimants drew benefits equalling 40–49 per cent of their wages.

6. The weekly benefit covered only 58 per cent or less of average weekly expenditures of most single claimants and less than 45 per cent for the main wage earners in 4-person families. Benefits paid at this level fell far short of the goal of covering non-deferrable expenses of claimants, no matter how these might be defined. The study demonstrated that it is possible to obtain successfully the kind of data needed in evaluating the unemployment insurance program in terms of meeting its objectives.

Adjustments of Families to Income Reverses. DOROTHY BRADY and PAUL KERSCHBAUM, *Bureau of Labor Statistics.*

In considering the question of how families adjust to income reverses, it may be well to note that income is disposed of in one of two ways. It is either expended for goods and services or saved. When deprived of usual income, families must therefore in the long run either reduce expenditures or modify or eliminate savings. This, of course, may not occur in any particular year, but over time a balance between income and expenditures plus savings must be achieved.

The pattern of response of families to income deprivation is affected by many factors, some of which provide an opportunity for free choice while others are inherent in the structure of society, or the family composition, or arise out of the social conditions prevailing in the period during which income reverses are sustained.

Expenditures studies have generally demonstrated that expenditure patterns do not change abruptly when income is reduced. Habit, imitative behavior, expectations of future enhancement of income, the degree to which income losses are widespread in the community or in a population group, and numerous other factors have been identified as forces which affect the manner and degree to which expenditure levels and patterns are altered.

The Railroad Outlook 1956. PIERRE R. BRETEY, *Hayden Stone & Co.*

The railroad shares have, price-wise gone far in discounting the fundamental improvements in this industry as measured by debt reduction, reduction of fixed charges and improved operating efficiency. Any further material advance in the stock market prices for railroad shares must depend upon altered views of institutions in their buying of these shares. They may well decide to buy these shares as investments for their liberal yields which offer one and half per cent more than the average of most industrial stocks.

Such institutional interest may be delayed until the Interstate Commerce Commission takes steps in freeing the industry from harmful regulation, in which case railroad securities my add another "leg" to their phenomenal advance as recorded in the past decade.

The Effects of Sexual Motivation on Phantasy. RUSSELL A. CLARK.

This paper involved the projective measurement of experimentally induced levels of sex motivation. Sexual motivation was aroused by the presentation of slides of nude females in two instances and in another instance by the presence of an attractive female. The TAT protocols were analyzed for manifest sex imagery and sex-involved guilt. The results showed that under normal conditions the experimental groups expressed significantly less sex and guilt in the TAT stories than did the control groups. Under conditions of alcohol these results were just the reverse. That is, the experimental group showed significantly more sex and guilt than did the control group. These results are interpreted by assuming that under normal conditions the guilt evoked by sexual arousal is sufficient to inhibit the expression of sex with a consequent lowering of guilt. Under alcohol, however, the guilt over sexual arousal is reduced enough to permit the expression of sex with a resulting increase in expressed guilt.

In addition this study involved scoring TAT stories for the presence of sexual symbolism. The stories scored came from two of the experiments described above. In one, the experimental group was sexually stimulated before taking the TAT, while the control group received no such prior stimulation. The second experiment was conducted in the same fashion except that it was carried out in a beer party atmosphere under the influence of alcohol. The results showed that under nonalcoholic conditions the aroused group expressed significantly more symbolism than the control group. Of course it had been shown previously that this aroused group inhibited the manifest expression of sex. Under conditions of alochol both groups gave very little symbolism. It had previously been shown that these two groups expressed large amounts of manifest sexuality. These results are, of course, in line with the Freudian hypothesis that inhibited sexuality finds an outlet symbolically in fantasy. However, a second, intragroup, analysis showing the amount of symbolism, expressed as a function of the amount of manifest sex expressed, revealed the relationship to be curvilinear with both low and high manifest sex corresponding to high symbolism. Various interpretations are discussed for this latter finding.

How the Retailer Uses Census Tracts. SIDNEY COHEN. *The Hecht Co.*

The retailer can use census tract data in many ways but essentially, tract information helps the retailer to know his market better and thereby aiding him in providing his customers with the proper goods and services. Several illustrations are given of possible applications. In the selection of branch store sites data, dwelling units by census tracts permits the retailer to better organize sales promotion efforts while special tabulations from the census bureau dealing with type of business breakdowns show the distribution of sales throughout the metropolitan area and are of value in planning new outlets. Census tract information is of use in establishing delivery zones because census tracts provide material to equate the size of delivery zones. The use of tracts also makes for flexibility of the delivery zones which is essential if the delivery manager is to be able to shift the daily overflow from overburdened routes to routes with small loads. Statistics on the age of the tract populations can prove helpful to retailers selling to specific age groups. The tracts also provide a good basis for analyzing the sales of merchandise by area and income groupings. Thus, differences in color, style and type of goods bought can be obtained. As a guide to charge account solicitation and credit needs, census tracts can perform a useful function. A continuing analysis of residence patterns of customers by census tract can establish the marketing area of a store and changes from the established pattern can be used to evaluate the effect of competitive outlets and the effectiveness of sales promotion efforts.

Statement of Criticisms of Industry Data and Summary of Changes Incorporated in the Presentation of Industry Data in the 1954 Census of Manufacturers. MAXWELL R. CONKLIN, *Bureau of the Census.*

The Census of Manufactures for over a century has provided industry data that has been used effectively for a great many purposes. For many of these uses the data as tabulated and published have no serious limitations. For some industries, however, these data are subject to potential misuse when input and ouput measures are compared because of the production of secondary products and variations in the extent of integration within individual establishments.

These difficulties are necessarily associated with the system by which establishments are classified. A more flexible approach to the rearrangement of the Standard Industrial Classification for special studies is desirable. Part of the questions raised by data users could have been answered by the more thorough use of published information. Special tabulations of data, though not as extensive as they should have been, have also been helpful and further tabulations of the 1954 census would be especially beneficial. Recent analyses of peace-time figures would seem to indicate that secondary production and changes in degree of integration have had less effect on changes in input-output relationships than has been assumed by some analysts. This needs more thorough study. The Census Bureau for many decades has been aware of industry data limitations and has taken steps to supply necessary additional information to the extent that its resources would permit. Without doubt, additional data on processes and

materials consumption are needed but the Census Bureau must have the help of statisticians and economists in industry, government, and research groups if significant further progress is to be forthcoming.

Statistical Aspects of Safety Testing Salk Poliomyelitis Vaccine. JEROME CORNFIELD, MAX HALPERIN, and FELIX MOORE, *National Institutes of Health.*

The history and problems of safety testing poliomyelitis vaccine are outlined. This includes the nature of the virus inactivation process and its relation to safety and the adequacy of the monkey and tissue culture safety test before and after changes in minimum testing requirements. Theoretical and empirical considerations show the assumption of linearity of inactivation is not valid and in itself cannot insure safety. Theoretical considerations show that the monkey test is not a practical safety screen. Using a Poisson model for tissue culture safety testing, which agrees well with available evidence, it is shown that test size initially required implied differential lot-size dependent sensitivity. It is then shown that present testing requirements impose essentially the same safety requirements on all producers. The question of "consistency," i.e., the ability of manufacturers to produce a uniform vaccine, is also considered. It is shown how testing experience yields upper bounds on the average infectivity of marketed vaccine and how these upper bounds can be used in a scheme insuring acceptable quality of outgoing product. Finally comparisons of poliomyelitis attack rates and tissue culture infectious units, based on lots of known infectious Cutter vaccine, are presented.

Special Foreign Programs for Teaching Experimental Statistics to Scientists. GERTRUDE M. COX, *North Carolina State College.*

Official statistics of a descriptive nature concerning the affairs of a nation is introduced first. The country's competitive and internal development depends upon such facts and figures. The second stage of statistical maturity frequently deals with the teaching of theoretical statistics. Later comes the third stage where the statistical methods such as sample survey, significance testing, estimation and experimental designs are such that the research workers, technologists and administrators frequently need to use these statistical techniques. Brief descriptions were given of the status of statistics in (1) Latin American countries, (2)Far East, (3) Near and Middle East, (4) Europe and Scandinavian countries, and (5) Western Pacific region.

The Work of the Inter-American Statistical Institute in the Field of Statistical Education and Training. O. A. DE MORAES, *Inter-American Statistical Institute.*

After an introductory description of the Inter-American Statistical Institute (IASI)—what it *is* and what it *does* —as a private non-profit association of statisticians of the American nations operating at the level of governmental organizations in the Western Hemisphere, the paper explains the Institute's activities in the specific field of statistical education and training. As significant activities developed by the IASI, the following undertakings are separately described with respect to their purpose and scope: (1) The continuous compilation of equivalent statistical terms in the four languages of the region—English, French, Portuguese and Spanish—resulting in the publication of a *Statistical Vocabulary*, the second edition of which is now in preparation; (2) the publication of the *Inter-American Textbook Series on Theoretical and Applied Statistics*, through which texts on a variety of statistical subjects are made available in the various languages of the Western Hemisphere to teachers of statistics, students and technicians; (3) the holding of short-term courses on some specific subject, as the "Inter-American Course in Statistical Sampling" held in Washington, D.C. in 1951; (4) the organization and administration, in collaboration with a sponsoring Government, of Inter-American training centers under the Program of Technical Cooperation of the Organization of American States, as the one on economic and financial statistics which was opened in Chile in 1953 and another on statistical sampling, the organization of which is now in its preliminary stages; (5) the promotion of regional centers for basic training in statistics, under the principal responsibility of a Government of the region and on a rotating basis from country to country within the area, the first of which, for Central America, was established in El Salvador (1954) and the second of which, for the Caribbean area, is pending final arrangments; (6) close cooperation in inter-American statistical training programs promoted by other international or national agencies, such as the training centers held in connection with the taking of the 1950 Census of the Americas in Mexico (1948) and Guatemala (1949), the "Inter-American Seminar on Biostatistics" held in Chile (1950), and the "Inter-American Centre on Biostatistics" which has been operating in Chile for the past three years. Principal problems faced in carrying out the activities mentioned above, the experience gained in their execution, as well as some results of the Institute's work in the field of statistical education and training as a whole, are presented and discussed in the final part of the paper.

A Survey of Fifty Countries' Industrial Census Programs, 1946–55. CLAIRE P. DOBLIN, *United Nations.*

During the last ten years, nearly twenty countries, most of them under-developed, took their first industrial censuses. Many of these censuses benefited from technical assistance, provided under the Point 4 program and administered by United Nations experts. At the same time, countries with more fully developed industries reappraised and expanded established census programs. As a guide to this unprecedented wealth of census material this paper analyzes the items of data most commonly sought by 50 governments in their recent censuses. This is followed by a grouping of the censuses into three types of frames, based on the major purposes for which the inquiries were taken, viz., to gather information on: (1) structure and resources employed by the manufacturing sector; (2) on resources and activities of the manufacturing sector; (3) a combination of the two purposes. The paper finishes with an outlook on future census programs.

The Oil Trade in the International Balance of Payments in 1951. CORNELIUS J. DWYER, *National Bureau of Economic Research.*

The landed value of Free World oil trade in 1951 was $7,260 million according to the paper's estimates, first to be published. Of this total, U.S. oil companies sold 57 per cent, British and British-Dutch, 39 per cent, and other companies 4 per cent.

The U. S. Balance of Payments on Oil: (millions of dollars)

Receipts		*Payments*	
U. S. Exports	$ 760	U. S. Imports	$ 550
Foreign sales of oil	2,140	Foreign purchases (for resale)	2,140
Merchandise exports to oil source countries	650	Dividends	40
Tanker freight	440		
Investment Earnings	900		
Total	$4,890	Total	$3,100
	Net Receipts $1,790		

The U. K. Balance of Payments on Oil: (first published estimate)

Receipts		*Payments*	
U. K. Exports	$ 100	U. K. Imports	$ 570
Foreign sales of oil	1,690	Foreign purchases (for resale)	1,690
Tanker freight	390	Tankers	210
Merchandise exports to oil source countries	240	Dividends	60
Investment earnings	720		
Total	$3,140	Total	$2,530
	Net Receipts $610		

The study was based on the customs records of 77 importing countries and 16 exporting countries. Using importers' figures for nonreporting Middle East oil exporters, importers reported receiving 244 million metric tons against exporters' shipments of 276. The difference of 32 million (worth an estimated $679 million, f.o.b.) consisted of military liftings, supplies for in-bond bunkering of foreign flag vessels, oil in transit and in bonded storage, etc.

1951, a difficult year because of mid-year nationalization of Anglo-Iranian Oil company and consequent trade dislocations, was chosen because it was the closest year for which full data were available when study commenced. Work on 1950, 1952, 1953 and 1954 is under way now.

Educational Aspects of In-Service Training in Statistics. R. PARKER EASTWOOD, *Columbia University.*

The segments in the title of this paper are defined as follows: (1) "educational aspects" relate to the lessons we should learn from the giving of in-service training in statistics; (2) "in-service training" is broadly interpreted to cover not only in-plant training but any other training received by employees on the recommendation of the employer and paid for by him; and (3) "statistics" is declared to include not only the training so labeled but much that is offered under such titles as "operations research" and "applied mathematics." The content of the course is taken as the criterion for classifying a course rather than the title under which it may be offered. The fact that industry moved quickly and expertly to provide training in statistics during the critical period of World War II stands as an object lesson to colleges and universities. Several groups associated with collegiate training must share the blame for institutions of higher learning's having failed to provide adequately trained personnel in the emergency. Teachers of statistics must bear the greatest responsibility, however, since they represent the first line of defense against statistical illiteracy.

Among the weaknesses in statistics instruction are the following: (1) required statistics courses that demand too little from the undergraduate student and given too late in their undergraduate program of study; (2) too many teachers of statistics who either are reluctant to teach statistics or have had little experience in teaching; and (3) too great emphasis upon statistics as a branch of mathematics and not enough stress on its manifold uses and applications together with the unifying theory of inductive inference. The most important pedagogical lesson that statistics teachers should learn from in-service training is the importance of visual aids and experimentation as a means of introducing the subject, particularly to students who need a knowledge of basic statistics but do not intend to become professional statisticians. Industry for its part should learn to make long-range plans for in-service statistics training with emphasis on the specific "known-how" of applications peculiar to a company or to an industry. The teaching of the "know-why" represented by principles and theory of statistics is the primary responsibility of our colleges and universities and should remain there. Among the current trends and future developments in teaching statistics, the most encouraging is the gradual reduction of inter-disciplinary barriers. The growing ease with which statisticians converse with one another is a good omen for the future growth of statistical knowledge. Communications, however, between statisticians and beginning students of the subject still remains a serious bottleneck. Greater progress toward a standardized system of symbols would aid greatly in reaching the best potential market for statistical training, both inservice and on-campus.

Maximum Likelihood Estimators of the Parameters of a Trivariate Normal Population from Incomplete Data. GEORGE L. EDGETT, *Queen's University.*

A random sample of size N is drawn from the trivariate normal population. n of the items are observed with respect to X_1, X_2, X_3 and n_1 with respect to X_1 and X_2 only, hence $N = n + n_1$.

The maximum likelihood estimators of the nine parameters, m_{11}, m_{21}, m_{31}, σ_{11}, σ_{21}, σ_{31}, σ_{12}, σ_{13}, σ_{23}, of the population are found.

Errors in Enrollment Projections: A Case History. JOHN K. FOLGER, *Southern Regional Education Board.*

Errors in enrollment projections made three years ago are studied by comparing them with actual enrollments since then. The error is broken into parts associated with each major assumption made. To the extent that the data

permit, the sources of error in the projections were studied, and the contribution of each assumption to the total error was measured. College enrollment projections had much larger errors than projections of public school enrollment. Part of this is because there is more individual choice associated with college attendance than with elementary and secondary school attendance, and part of the larger error is caused by the less comprehensive enrollment statistics available at the college level. The analysis of errors revealed that the methods used consistently underestimated college enrollment during the past three years. No one source of error was largest or smallest in every year. Improvements need to be made in estimating the number entering college each year, the number dropping out, and the number graduating each year. Such improvements will depend on the collection of additional data so that a more complex estimating procedure can be used. Present estimating procedures have to make too many simplifying assumptions because the necessary enrollment data is not available at state or national levels.

On the Consistency of Subjective Ratings. J. E. FREUND and A. N. POLNER, *Virginia Polytechnic Institute.*

If a number of judges rate a product subjectively it is not only important to know the average of their ratings, but it is often essential to evaluate their consistency. The method proposed here is based on restricted occupancy theory and it is actually a test whether the total number of points assigned to a product by a set of judges is randomly distributed. Formulas and tables are provided for calculating moments of the test statistic.

Errors Associated with Process Adjustments. RICHARD A. FREUND, WILLIAM G. HOWE and J. EDWARD JACKSON, *Eastman Kodak Company.*

In addition to being concerned with risks of Type I and Type II errors, many industrial people must consider the errors of over and under correction when making process adjustments. After determining that a process is "out of control" it is necessary to estimate the actual deviation from standard in order to make the required corrections. When measurement variability, etc., is large, the risks of incorrect adjustments become a serious problem. This paper includes a discussion concerning the risks of errors associated with process adjustments and an example showing how an analysis of these risks helps determine a reasonable operational procedure.

The Introduction of Risk into a Programming Problem. RUDOLF J. FREUND, *Virginia Polytechnic Institute.*

Experience has shown that optimum programs of some firms rely heavily on some high risk enterprises. This is due to the fact that high risk enterprises have high mean revenue together with a high variance; the data for programming involves only the mean revenue. The variance of revenue may be incorporated into a programming model by the use of the expected utility hypothesis. Under certain assumptions this results in a quadratic programming problem.

Wharton School Study of Consumer Expenditures. IRWIN FRIEND, *University of Pennsylvania.*

The Study of Consumer Expenditures, Incomes and Savings by the Wharton School in cooperation with the Bureau of Labor Statistics under a Ford Foundation grant is divided into two parts: first, a series of eighteen volumes of tabulations of expenditures and related data for 1950–1951 compiled by the B.L.S. from surveys covering 1,500 items of budget information collected from 12,500 families and individuals in 91 cities; second, a series of research studies of economic and social aspects of consumption and saving patterns, by staff members of the Wharton School and other academic institutions, making use of the new data, as well as of other consumer surveys and time series material.

Some preliminary findings follow: entrepreneurs accounted for a large proportion of aggregate personal saving, having appreciably lower expenditures than others in middle and upper income groups on most consumption items; homeowners, particularly those who bought homes in 1946–49, consumed less than renters reflecting lower expenditures not only on housing but on other expenditure categories; families in small cities spent less of their income than those in large cities especially for food; the aged spent less, particularly for clothing and recreation, than others in the same income class; manual workers spent somewhat more on food and less on housing and household operation than others.

Summary of Report of Federal Reserve Consultant Committee on Business Plant and Equipment Expenditure Expectations. IRWIN FRIEND, *University of Pennsylvania.*

The Federal Reserve Committee on Plant and Equipment Expectations attempted to appraise the predictive and analytical value of data on business expectations, plans, decisions, and commitments to purchase or construct new plant and equipment. Two types of recommendations were made: first, to expand coverage, add further breakdowns, make more timely, or otherwise improve existing statistics whose anticipatory value seemed clearly indicated by all the tests which could readily be applied; second, to explore the untility of specific new series which appeared promising and to test further the value of existing statistics. It was the Committee's judgment that short-term projections of plant and equipment expenditures based on survey data have given better results than alternative procedures, such as extending current expenditures, adjusting current expenditures by the rate of change in such outlays or estimating prospective outlays from lagged profits. There is some evidence that the projection of plant and equipment expenditures might be improved by adding as an explanatory variable the difference between actual and anticipated sales. Eventually it may be feasible to insert other operational variables in addition to sales in these relationships. I would like to add the observation that the case for plant and equipment anticipations as a forecasting device seems to rest on a more secure empirical basis than for other types of business expectations.

Stock Market Outlook: Retail Trade. J. M. GALANIS, *R. L. Day and Co.*

Aside from certain of the food chains concentrating on supermarkets and several of the leading department store chain issues, retailing issues are a drab lot. Variety store chains, with their alarming decline in profit margins post-war have lost their former luster as defensive, income-stability holdings. Gross-competition has extended to

most retailing divisions and made for stiffer competitive conditions. Exodus of population to the suburbs, especially the upper and middle income groups, has impelled the rise of huge shopping centers and food supermarkets. The best looking division of retailing appears to be the department store chain field, where several good investment quality equities are available at moderate price-earnings ratios and good yields of 5 per cent or better from well-covered dividend rates with record levels of disposable consumer incomes and 66 million employed at historically high wage rates, further going in sales, earnings, and in some cases, dividends, loom for 1956. Suburban populations are becoming large enough to support really big, complete department stores, instead of earlier small branches. Store managements are very expansion minded. Federated, May, Allied, Associated, and M. Field are particularly interesting values.

Theoretical Considerations Incident to the Construction of an Uphill Truck Lane or Passing Bay in a Mountainous Area. JAMES P. GEORGE, *C. B. Caldwell Associates.*

At what distance from the bottom of the grade should the construction of an additional lane or passing bay begin? At what distance should such a facility terminate? Upon first thought one may be inclined to the belief that this is a problem for the highway designer or engineer. The highway engineer has been tardy in recognizing that statistical method in his most useful tool. He should fully realize the importance of this method of approach in the solution of his problems. On Highway 60 at Quartzsite, Arizona, markers were erected at intervals of one-tenth of a mile. Beginning at the bottom of the grade, the first market was designated as Station 0, the second marker as Station 1, and so on up the grade until the distance of one mile had been covered, and the tenth marker or Station 9 had been reached. Workers with field glasses and stop watches were stationed within easy observing distance of the roadway under investigation. Records were kept, indicating by class intervals, the varying numbers of trucks or commercial vehicles responsible for retarding the movement of otherwise free moving vehicles, traveling in the same direction. Also recorded was delay time in seconds.

Quantitative Measurements of the Intermetropolitan Wage Structure. WILLIAM GOLDNER, *Bowling Green State University.*

The intermetropolitan wage structure, which is made up of the wage levels of standard metropolitan areas (SMA's), reflects a variety of influences. Using BLS community wage survey data and Census statistics, quantitative tests of several models reveal that the most important factors associated with metropolitan wage levels are (1) the degree of unionization in the SMA, (2) the population of the SMA, and (3) the ratio of the median rural family income in the area surrounding the SMA to the median urban family income within the SMA. Variations in industrial structure which inferentially are presumed to affect the wage level do not have any significant influence in large and mature SMA's. Differentials between skilled and unskilled wage levels in individual SMA's remain constant for differing degrees of unionization, signifying that wage changes follow across-the-board absolute patterns of increase in response to unionization. On the other hand, skilled wage levels respond at twice the rate that unskilled wage levels do to differences in SMA population. Larger SMA sizes therefore widen skill differentials. Finally, unskilled wage levels respond substantially to differences in rural-urban income ratios at the same time that skilled wage levels are unaffected, narrowing skill differentials.

Problems in Projecting Enrollments in Institutions of Higher Education. HAROLD GOLDSTEIN, *Department of Labor.*

The U.S. may be in the midst of an "educational revolution" which will result in extension of college education to a broader segment of the American people. Because college-age population is growing and a larger proportion of them are going to college, in 15 years we may see 6 million or more students enrolled in colleges, more than double the present number. Such a revolution has already taken place in the high schools. In a thirty-year period before World War II, the proportion of youngsters enrolled in high school shot up from 15 per cent to 75 per cent; in a single generation the high school changed from a college preparatory academy for the few to the minimum standard education for all young people.

Just before World War II, the colleges had reached the point where the high schools were 30 years earlier, at the threshold of their great expansion; college enrollments in 1940 amounted to 15 per cent of the population of college age. Since then the proportion of young people going to college has increased phenomenally. In the 15 years since 1940 the per cent of the 18-year-olds entering college doubled—from 16 per cent then to 32 per cent last fall. Factors behind this increase include: (1) higher family income levels and greater security; (2) widely publicized shortages of engineers, scientists and other highly trained workers; (3) the veterans education program; (4) public recognition of the value of college education—expressed, for example in the student deferment program during the Korean crisis; (5) accessibility of college education to more people through "community colleges" (or junior colleges); (6) most recently, increasing concern lest we fall beind the "iron curtain" countries in our supply of highly-trained manpower.

Extension of college education to a higher proportion of young people is likely to continue. The population of college age will begin growing rapidly by the mid-1960's and by 1970 the number of people aged 18–21 will be 14¼ million, compared to 8¼ million in 1955.

A Note on Uniform Best Unbiased Estimators for Variance Components. FRANKLIN GRAYBILL and A. W. WORTHAM, *Oklahoma Agricultural and Mechanical College.*

Consider the linear model given by

$$Y_{n_1 n_2 \cdots n_k} = \mu + A_{n_1}^{(1)} + A_{n_2}^{(2)} + A_{n_1 n_2}^{(3)} + \cdots + e_{n_1 n_2 \cdots n_k} \tag{1}$$

where $Y_{n_1 n_2 \cdots n_k}$ is the observation, μ is a fixed unknown constant generally called the overall mean, the $A_{n_1}^{(1)}$, $A_{n_2}^{(2)}, \cdots, e_{n_1 n_2 \cdots n_k}$ are independent normal variables whose means are zero and whose variances are $\sigma_1^2, \sigma_2^2, \cdots,$

σ_i^2 respectively (σ_i^2 distinct). This model is commonly called a component of variance model and sometimes referred to as the Eisenhart Model II [4]. For example, if the model is a two-way balanced classification, it can be written as

$$Y_{ij} = \mu + a_i + b_j + e_{ij} \qquad i = 1, 2, \cdots, m_1; j = 1, 2, \cdots, m_2 \tag{2}$$

where the a_i, b_j, and e_{ij} are independent normal variables whose means are zero and whose variances are σ_1^2, σ_2^2, and σ_3^2 respectively, and corresponding to the notation in (1) it follows that $a_i = A_{a_i}^{(1)}$ and $A_{a_j}^{(2)} = b_j$.

In models such as the ones above, it is desired to estimate the σ_i^2 or linear combinations of the σ_i^2. This can be done by partitioning the corrected total sum of squares $\Sigma(Y_{a_1 a_2 \cdots a_k} - y_{\cdots})^2$ into t non-negative quadratic forms $Q_1, Q_2, \cdots, Q_t$ such that (where $y_{\cdots}$ is the mean of $Y_{a_1 a_2 \cdots a_k}$)

$$\Sigma(Y_{a_1 a_2 \cdots a_k} - y_{\cdots})^2 = \sum_{j=1}^{t} Q_j. \tag{3}$$

If f_j represents the degrees of freedom associated with the sum of squares Q_j, and if L_j represents the expected value of Q_j/f_j, then L_j is a linear function of the σ_i^2.

If the t equations

$$\frac{Q_j}{f_j} = L_j \qquad = 1, 2, \cdots, t \tag{4}$$

are solved for the σ_i^2 (let $\widehat{\sigma}_i^2$ represent the solution; $i = 1, 2, \cdots, t$), then these values are the analysis of variance estimates of the corresponding variance components σ_i^2.

In this note conditions are stated which are sufficient for the solutions to the system of equations in (4) to be uniformly best (minimum variance) unbiased estimators of the variance components, σ_i^2, for any size of sample.

Measurement of Current Trends in Output per Man-Hour. LEON GREENBERG, JACK ALTERMAN, and ALLAN D. SEARLE, *Bureau of Labor Statistics.*

The Bureau of Labor Statistics released in December 1955 four types of indexes of output per man-hour in manufacturing, which showed average annual increases ranging between 3.0 and 3.6 per cent from 1947 to 1953.

General interest in more current information led the BLS to experiment with several other measures for estimating 1953–55 trends. In part this research was to develop measures which would avoid the use of a built-in productivity assumption, as in the total FRB production index in current years (about 50 per cent). The Federal Reserve Board indexes of production for total manufacturing and selected industries, Office of Business Economics statistics on sales, and BLS price deflators and man-hours were used to compute various experimental productivity measures, which were compared with the official BLS series.

Figures are available for too few years to judge whether these experimental measures are reliable indicators of current trends, although collectively they show promise of being useful. For the two year span 1953 to 1955 the experimental measures suggest a productivity increase in the neighborhood of 10 per cent.

Financial Intermediaries and the Saving-Investment Process. JOHN G. GURLEY, *Univ. of Maryland*, and E. S SHAW, *Stanford University.*

The security issues of nonfinancial spending units (primary issues) are related to the units' aggregate deficit on income and product account, and the deficit in turn is related to the level of GNP. Ordinarily, at given interest rates, spending units can be expected to reject some portion of the issues emerging at any income level, and demand the debt issues of financial intermediaries (indirect assets) instead. This incremental demand for indirect assets is in part a demand for portfolio diversification. If income is rising, too, there is an incremental demand for money and perhaps for other indirect assets for transactions and contingency balances. To the extent that the issues of nonmonetary intermediaries are competitive with money balances of whatever type, the expansion of these intermediaries reduces the growth of the monetary system that is required to hold yields constant.

The movement of long-term interest rates from 1900 to 1930, when these rates are averaged over complete cycles, was closely related to the level of the indirect (or direct) finance ratio. This ratio is the proportion of primary issues absorbed by financial intermediaries, including the monetary system. A high indirect finance ratio lowered long-term rates, and a low ratio raised them. This relationship was much closer than that between the income velocity of money and long-term rates.

Energy Consumption and International Comparisons of Technological Development and Economic Change. NATHANIEL B. GUYOL.

There is a sound theoretical basis for believing that energy consumption data can be used in the measurement of certain economic and technological phenomena. Investigators who have looked into this in the past found that, while energy consumption was related to such things as national product and industrial production, the relationship was not close. The findings of these investigators were based, however, on gross or net consumption of energy, rather than *Effective* consumption. More recent studies, of industry and of the total economy, indicate that there may be a fairly close relationship between *effective* energy consumption and output. If this conclusion is borne out by further studies, energy consumption may prove to be an extremely useful tool for the measurement of production and productivity, in the economy as a whole and in various sectors of the economy, especially at the international level.

A Test of Longevity of Two Types of Hypodermic Syringes Conducted under Operational Conditions. BERTRAM W. HAINES, *Johns Hopkins University.*

In this paper, there is presented a discussion of a study conducted in Harriet Lane Home of The Johns Hopkins Hospital designed to determine the cost of using each of two types of hypodermic syringe. The two types are glass

tip and non-locking metal tip. Consideration is given to problems of finding a suitable place in the hospital to conduct the study. Once the place is chosen, the design of the study is discussed. The conduct of the study and the data collected are described. The analysis of the data showed that The Johns Hopkins Hospital may save as much as $730.00 per year by supplying metal tip instead of glass tip syringes to Harriet Lane Home.

Epidemiologic Evaluation of the 1955 Polio Vaccine Program. WM. JACKSON HALL, *Public Health Service, Atlanta, Ga.*

A summary account is given of the activities of the Poliomyelitis Surveillance Unit, Communicable Disease Center, Public Health Service, in evaluating the use of poliomyelitis vaccine in the nation this year, as described in "The Surveillance of Poliomyelitis in the United States in 1955," a paper presented at the American Public Health Association meeting in Kansas City in November. The statistical aspects of this paper are elaborated.

The Polio Surveillance Program has been concerned with two main problems: the epidemiologic evaluation of the safety of polio vaccines and the measurement of their effectiveness. An epidemiologic study was made of cases of polio which occurred following inoculations with Cutter vaccine, and it was concluded that the development of the disease, in some of these cases, was the result of the presence, in infectious amounts, of live polio virus in some distribution lots of Cutter vaccine. No evidence has come to light to incriminate any lot of vaccine released since safety tests were revised in May, 1955. Preliminary reports of special studies in a number of states indicate encouraging results regarding the effectiveness of the vaccine as used in the United States in 1955. Attack rates for paralytic polio are from two to more than five times greater in the unvaccinated than in the vaccinated groups of children. Confirmation of these preliminary findings has been obtained from a study of the pattern of the age distribution of cases of poliomyelitis reported this year in a number of states. A sharp reduction in paralytic attack rates in 7- and 8-year-old children has been observed, in comparison to the expected rates based on past experience. This finding constitutes an independent confirmation of the effectiveness of the vaccine as used this year.

Smoking and Cancer: Consideration of Some Statistical Aspects. E. CUYLER HAMMOND, *Yale University.*

The subject of smoking and lung cancer is of no small importance. It must be borne in mind that an erroneous conclusion in one direction could cause unnecessary financial loss for many people. An erroneous conclusion in the other direction—or delay in arriving at the correct conclusion—could result in tens of thousands of people unnecessarily developing and dying of lung cancer. Perhaps we should particularly bear in mind the influence which the scientific conclusions might have on children who have not yet started to smoke. No less than 17 independent studies have been carried out in the United States, England, Germany, Finland, and the Netherlands to determine whether smoking is associated with lung cancer. Each one of them showed a much higher percentage of smokers among lung cancer patients than among people without lung cancer. Statisticians and physicians of the highest repute were included among the investigators.

These findings have been confirmed by two large scale studies in which men were questioned about their smoking habits and then followed for a number of years. One of these studies was made by Richard Doll and A. Bradford Hill in England. They have reported on the death rate of 24,389 British doctors, followed 20 months after they answered a smoking questionnaire in 1951. The lung cancer death rate increased sharply with the amount of smoking. The factors of occupation and social economic status could not have biased the findings since all of the subjects were of the same profession.

E. Cyler Hammond and Daniel Horn have studied the death rate among 188,078 white American men between the ages of 50 and 69. These men were questioned about their smoking habits in the early part of 1952 and traced through October 31, 1954. The lung cancer death rate was extremely low among nonsmokers but high among heavy cigarette smokers. This was found for men living in rural areas as well as for men living in cities.

Last summer Joseph Berkson discussed the first published report of the last named study. The published report covered only the period through October 31, 1953. He noted that the death rates for this period were very low. He then suggested that the findings on smoking might have been due to proportionally more well and fewer ill nonsmokers than smokers being included in the study.

Dr. Hammond pointed out that the study was specifically designed to avoid this type of difficulty. For this reason, obviously ill men were excluded in selecting the subjects in 1952. Lung cancer has such a high case fatality rate that most of the few lung cancer patients inadvertently included in the sample would have died in about a year. Thus any possible bias of the type suggested by Berkson was eliminated after about that length of time. Figures for later periods show an even higher association between smoking and lung cancer death rates than figures for the first period.

In the opinion of the author, there is overwhelming evidence that cigarette smoking increases the risk of lung cancer. This does not imply that smoking is the only factor involved.

A Game Learning Model of Interaction. DAVID G. HAYS, *The RAND Corporation.*

The classic theory of games is normative and static. Sociological theory may include a dynamic, descriptivt theory of games as a model of interaction. This paper describes an experiment and several analyses which suggest lines of development for such a theory. Two subjects participate in one hundred trials of a two-choice experiment, similar to that of L. G. Humphreys. One of two events occurs with probability .75. The two subjects, as a group, are assigned a score for each trial, based on the number (0, 1 or 2) of correct predictions of the event. Summary statistics show that the behavior of the subjects depends on the score function and on the behavior of the other subject in the group. A description of one trial in the form of an extended game is given. The behavior of each subject is assumed to be governed by a set of probabilities described as his strategy for the game. These strategies are modified from trial to trial in accordance with the outcomes of the trials. If the strategies of the subject are not all identical, the initial distribution of strategies is examined by methods discussed. A stochastic process of the form introduced by Bush and Mosteller is applied to describe the modifications of strategies. An additive approximation previously developed by Mosteller is attempted here. The use of stochastic matrices, describing change of state from trial to trial, when the states are trial outcomes, is described, and some results are presented.

Financing Higher Education in the United States. ERNEST HOLLIS, *Office of Education.*

Altogether the 1,857 colleges and universities of the United States have approximately $5 billion of endowment that has been accumulated since colonial days, the income from which may be used for either capital or current support purposes. These nonexpendable funds are being added to at the annual rate of $250 million. The plant facilities of institutions of higher education in this country are carried on their books at what it cost to construct the facilities. On this basis of valuation the college plants of the United States are worth $7 billion. The nation now has an estimated deficit in plant facilities of at least $6 billion, and an expansion need during the next 10 years of at least $7 billion. The backlog is accounted for by obsolescent structures that are more than 50 years old and by temporary facilities provided from war reuse structures at the close of World War II. Approximately 15 per cent of the square footage of buildings available for education purposes at the colleges and universities of the country are of a temporary character. Approximately 60 per cent of the new college construction that has been erected in each year since 1950 was paid for from tax sources and 40 per cent of it from the gifts and bequests of individuals and corporations. A recent survey conducted by the Council on Financial Aid to Education indicates that 30 per cent of the corporate gifts and 40 per cent of the individual gifts of philanthropically inclined corporations and individuals is dedicated to capital and endowment purposes.

The annual budgets of our 1,857 colleges and universities aggregate $2.5 billion, of which slightly more than $2 billion is dedicated to educational and general administrative purposes. Approximately 54 per cent of these current budgets are at publicly and tax-supported institutions and 46 per cent at privately controlled colleges and universities. The taxpayer is, of course, the largest provider of funds at tax-supported institutions with income from student tuition and fees ranking second in importance. At privately controlled institutions, student tuition and fees pay the largest single part of the operating budget with gifts and bequests from former students and the general public the next largest segment of income. Gifts from private business corporations are next in importance and constitute the most rapidly increasing source of support for privately controlled colleges and universities. Altogether private benefactions to institutions of higher education during 1954–55 approximated $500 million, which compares with $339 million for the school year 1951–52. Approximately 1/5 of the private benefactions go to publicly controlled institutions and 4/5 to privately controlled institutions. It is conservatively estimated that new private funds more than double current giving will be required for increases in enrollments during the next ten years. Funds estimated as needed in each of the next ten years is $608 million, of which $206 million would be for current budgets and $402 million for the expansion and replacement of capital facilities.

Cyclical Changes in Input-Output Relations. THOR HULTGREN, *National Bureau of Economic Research.*

Changes in the cost of doing business might help to explain recessions and revivals. The history of costs is being surveyed from this point of view at the National Bureau of Economic Research. One factor in cost is the amount of labor required per unit of product. In manufacturing, technological progress has dominated the relation of manhours to product; input per unit has tended to fall during expansions of output and during contractions alike. The percentage of industries with rising input per unit, however, it not as small in contractions as in expansions. In the absence of technical advances, input per unit in most of the manufacturing industries studied would probably fall during expansions but rise during contractions. In iron and copper mining, and on railroads, it has actually done so in spite of technical advances.

Input per unit could fall during most of an expansion, and rise during most of a contraction, and still reverse its direction of change toward the end of either. Such possibilities have been studied, so far, in four industries—railroads, meat packing, cement and steel. Each expansion or contraction is divided for this purpose into four consecutive segments. Reversals of direction were more common in the last segment than in others, but did not predominate even there. When cost did reverse its direction, the change was usually small in comparison with the opposite change that had already occurred. Input per unit was generally smaller at the end of an expansion than at the beginning, larger at the end of a contraction than at the beginning.

The Contraharmonic Mean $\Sigma X^2/\Sigma X$. NATHAN JASPEN, *National League for Nursing.*

Some properties of the contraharmonic mean, $\Sigma X^2/\Sigma X$, are presented. The standard error of the contraharmonic mean is also presented. Frequency distributions in social statistics often consist of aggregates such as cities, families, schools. Implicit in such distributions are distributions of individuals. Characteristics of the average individual are furnished by the arithmetic mean of the implicit distribution, which is the same as the contraharmonic mean of the explicit distribution. For instance, if the distribution consists of two families, of two children and six children respectively, the average child is a member of a family of five children. The average individual, in short, is not to be confounded with the individual in the average city, family, or school. The contraharmonic mean exceeds the arithmetic mean by a quantity equal to the sample variance divided by the arithmetic mean. The standard error of the contraharmonic mean can be shown to be equal to

$$\left[\frac{\Sigma X^4}{(\Sigma X)^2} - \frac{2\Sigma X^2 \Sigma X^3}{(\Sigma X)^3} + \frac{(\Sigma X^3)^2}{(\Sigma X)^4}\right]^{1/2}.$$

Census Program for Central Business Districts. HARVEY KAILIN, *Bureau of Census.*

A program for providing business statistics for the Central Business District (CBD) of large metropolitan centers recently has been adopted by the Bureau of the Census in connection with its 1954 Census of Business work. The immediate objective is to provide a basis for comparing changes in business activity in the CBD with those in the remainder of the central city and in the entire metropolitan area. The first CBD tabulations to be made will consist primarily of retail trade data from the 1948 Censuses of Business.

One provision of the program requires that the boundaries of the CBD be recommended to the Bureau by the local Census Tract Committees. At the present time CBD plans have been submitted by the Tract Committees in some 75 cities.

Because there is no standardized definition of the CBD nor generally accepted criteria for identifying the boundaries of the CBD, the Census Bureau, without attempting to provide a specific CBD definition, has merely described the CBD as "an area characterized by a high concentration of retail businesses, offices, theaters, hotels, and 'service' businesses and by high traffic flows." The Bureau's rules provide that the CBD ordinarily shall be defined to follow existing tract lines, though provision is made, subject to certain limitations, for altering tract boundaries where necessary to achieve a meaningful CBD.

Although the Census CBD program emphasizes the importance of keeping CBD boundaries constant in the interest of making statistical comparisons over time, the Bureau has recognized that the program is an experimental one and that future work on the CBD problem may require redefinition of the CBD's now being established in this program. The resolution of such problems as (1) what are the primary objectives to be met by defining CBD's and providing statistics for them; (2) should there be a standard definition of the CBD, which can be applied to all CBD's; (3) should the CBD boundaries be drawn to include or to exclude blocks which are predominantly occupied by wholesale or manufacturing activities; (4) should the tract boundary restriction be abandoned; (5) what specific criteria shall be used in identifying CBD-type area, e.g., property valuation, height of structure, traffic density, etc. are needed to guide future CBD work.

Weibull Distribution in Life-Testing of Electron Tubes. JOHN H. K. KAO, *Cornell University.*

The properties of the Weibull distribution are studied. The maximum likelihood estimates of the shape and scale parameters of this distribution for an ordered and truncated sample are found. A logarithmic transformation of the Weibull distribution function makes possible the least squares estimates of these same parameters. Both methods of estimating parameters assume the location parameter equal to zero. This is reasonable in the case of failure age data because the observations have non-negative values and the exposure to risk starts immediately with experimentation. Experience with several sets of failure age data consisting of a total of more than two thousand electron tubes fitted to Weibull distributions provides evidence for a common shape parameter. If the location and shape parameters are known, the maximum likelihood estimator of the scale parameter has statistically desirable properties; namely, sufficiency, unbiasedness, consistency and efficiency. Furthermore, with the location and shape parameters known, one may transform the failure age data to fit an exponential distribution and certain theories derived for the exponential case carry over immediately.

On Variances of Differences in Multi-Stage Samples. LESLIE KISH, *University of Michigan*, and IRENE HESS, *Washington, D. C.*

We discuss the estimated variances for statistics like $r = x/y$ and like $(r_1 - r_2)$ where each r is the ratio of two random variables x and y. The difference $(r_1 - r_2)$ occurs frequently and importantly in multi-stage samples as a) the change between two surveys of the same statistic, and as b) the comparison of the same characteristic for two subclasses from the same survey. Several differing computational forms for $\text{var}(r_1 - r_2) = \text{var}(r_1) + \text{var}(r_2) - 2\,\text{cov}(r_1 r_2)$ are exhibited. The effects of $\text{cov}(r_1 r_2)$, arising from use of identical sampling units for r_1 and r_2, are shown for a variety of survey data.

Frequency Analysis of Human Responses. EZRA S. KRENDEL, *Franklin Institute.*

Two types of man-generated time series have been investigated at the Franklin Institute. The first type of time series is generated when a man performs an eye-hand coordination, and the second type of time series arises from the electrical activity of the brain. Both of these types of time series have been studied in the frequency domain.

The particular setting in which eye-hand coordination was studied was the control of an F-80A in simulated flight. It was desired to obtain linearized transfer functions, or perhaps, "descriptive functions" which would describe the manner in which the pilot converted a visual input to a manual response. It can be shown that this linearized transfer function for open loop human control behavior is equal to the cross-spectrum of the control response and the forcing function divided by the cross-spectrum of the error function and the forcing function. The forcing function, which is crudely simulated rough air through which the pilot flies, is produced by filtering a low frequency Gaussian noise source at various low cut-off frequencies. The control response is the position of the pilot's control stick and the error function is the instantaneous position of the target on the visual display. Certain preliminary linearized transfer functions have been obtained. Electroencephalographic time series have been studied both by means of the spectra of the time series arising from the parieto-occipital and fronto-control lobes, as well as by means of cross spectra computed at dominant frequencies. Cross spectra between visual and auditory stimuli and corresponding evoked EEG potentials have been measured.

Methodology and Appraisal of Consumer Expenditure Studies. HELEN HUMES LAMALE, *Bureau of Labor Statistics.*

The value of consumer expenditure data depends on an understanding of what they are and a recognition of their limitations. Yet, relatively little research has been directed toward determining the nature and source of errors in the data. Three general approaches to appraisal of expenditure data are discussed, using the Bureau of Labor Statistics' Survey of Consumer Expenditures in 1950 as illustration. The three ways discussed are: (1) investigation of errors and limitations inherent in the survey procedures; (2) reconciliation of the data with similar data from different sources; and (3) examination of the internal consistency of the data and the reasonableness of the analytical results. In discussing appraisal by investigation of errors and limitations inherent in the survey, the methodology of the 1950 study is described briefly. This description points out the importance of the definition of the expenditure reporting unit used in these studies, distinguishing between the basic definition of the consumer unit and the time reference to which it is applied. It also summarizes the importance in these data of sampling errors, and errors of response and non-response. The discussion of appraisal by reconciliation of the data with similar data from different sources points out some of the problems in comparing consumer survey data with estimates of national aggregate expenditures. It also indicates some more limited comparisons with Census data and Bureau of Labor Statistics price reports. The discussion of appraisal by examination of the internal consistency and reasonableness of the analytical results

cites internal checks in the collection and processing of the data and recent analyses of the income-expenditure relationship. The conclusion indicates the responsibility of both producers and consumers of the data for improving the methodology and appraising the results.

The Position of Common Stocks Today. RICHARD W. LAMBOURNE, *Dodge and Cox.*

The general vitality of price action in recent weeks must be surprising to a great many people including those who were panicked into selling during the September-October decline. Further, it is well-known that some institutional investors have been reducing common stock holdings all year, especially in the buoyant markets of late summer. As far as we know the proceeds of these liquidations have been retained largely in cash or reinvested in fixed-income securities. In other instances portfolio managers have placed accruing cash funds in short-term governments, being concerned about the outlook for long-term bond prices as well as common stocks.

I submit that what actually happened was a fundamental *breakaway* from the abnormally low level of price-earnings multiples that had typified the American stock market since 1946. This reasoning, if correct, has profound significance. It would mean that common stocks have received a more lasting lift in basic valuation than is generally supposed. Among other things it would relegate to the scrap-heap a number of value theories based on price-earnings "ratios" of the postwar period.

The logical question at this juncture is whether price-earnings multiples are apt to go down again quickly in the future. I consider it most unlikely.

Flight Methods for Measuring Gust Spectra. U. O. LAPPI, *Cornell Aeronautical Laboratory, Inc.*

Two methods used to estimate turbulence or gust velocity spectra from aircraft response measurements are described. The older of the two methods is shown to require detailed information of aircraft dynamic behavior, which, in general, is difficult to specify accurately for gust disturbances. The newer method, based on an elementary relative velocity principle, does not require detailed dynamic behavior information. This method enables the motion of the air mass to be determined from instantaneous measurements made on the airplane. These simultaneous response measurements, recorded as continuous functions in turbulent air, are appropriately added to determine the instantaneous gust velocities. The gust velocities are then processed as random functions, using generalized harmonic analysis and sampling theory considerations, to determine gust spectral estimates. These estimates are shown for the vertical component of turbulence. A comparison of the two methods is made on the assumption that exact aircraft dynamic response information is available. Under this assumption, it is shown that the new method is more accurate in measuring the longer wavelength turbulence, but slightly less accurate for the shorter wavelengths. The new method is shown to be capable of providing such additional information as simultaneous gust measurements along the span of the airplane, and the possibility of defining, by flight test, dynamic aircraft response characteristics to gust inputs.

Sampling Distributions When the Sample is Drawn from a Finite Normal Population. ANDRE G. LAURENT, *Michigan State University.*

A random sample $\xi = (\xi_1, \cdots, \xi_i)$ of size i, is drawn from a finite population of N independent items $X = (X_1 \cdots X_N)$, with mean $\bar{X}$, and standard deviation S; the U_βs are normally distributed $N(m, \sigma)$ with unknown m or (and) σ.

The joint conditional distributions of $\xi_1 \cdots \xi_i$, given $\bar{X}$ or (and) S, and the joint distribution of $T = (t_1, \cdots, t_i)$ with $t_j = (\xi_j - \bar{X})/S$ are multivariate Beta distributions. Conditional distributions of the means $\bar{\xi}$ the variance S_ξ^2 and the second moment of the sample ξ are obtained, and the distributions of the same quantities and other statistics, for the sample T, are derived. The distribution of the second moment involves an expansion into hypergeometric functions. Some of these results generalise previous results of Irwin (1929) and Thompson (1935). Among possible applications, (some of them are outlined), a test is suggested to decide whether, in a set of samples which are supposed to come from the same population, a given sample is "significantly" different from the others.

Statistical Symposia for Biological Scientists. E. L. LECLERG, *Dept. of Agriculture.*

Relatively few biologists have had adequate formal courses in statistical procedures. As a result, many have a limited knowledge of statistics and experimental designs. The medium of joint symposia between the Biometric Society and various scientific societies is important in keeping biologists informed of the latest developments in statistical procedures. There is no question that this activity should be continued. As a result of a survey of 250 biologists of the State agricultural experiment stations and the United States Department of Agriculture, the following comments and suggestions should be given careful consideration in setting up joint symposia: (1) such symposia should be held more frequently and extended to other biological groups; (2) the greatest need is for programs presenting new procedures; (3) keep to a minimum the discussion of theoretical problems; (4) careful consideration should be given to the speakers; (5) use of illustrative material (handouts) is very important; (6) use of panel discussions may be practical at some meetings; (7) closer working relations with the officers and executive and program Committees of biological societies may be worthwhile in setting up programs; and (8) a questionnaire mailed to a limited portion of the membership of a society will develop subjects appropriate for programs.

The Use of Census Tract Data in Studying the Social Determinants of Juvenile Drug Use. ROBERT S. LEE, *New York University.*

As available information was contradictory and unreliable, this study was undertaken to discover the extent of adolescent drug use, its geographical distribution and the characteristics of high drug use neighborhoods. We collected the names and addresses of boys from the ages of 16 through 20 who had, in the four year period from 1949 to 1952, come to the attention of the courts and city hospitals in connection with narcotics. When duplicate cases were eliminated, we arrived at a figure of 1,844 boys who had become involved with the use, possession or sale of

drugs. There were, on the average, more than 500 new cases a year from 1950 to 1952. In addition, there was one girl involved with narcotics for approximately every five boys.

Maps were prepared which indicated that adolescent drug use was highly concentrated in a small number of areas—obviously the poorest sections of the city, inhabited predominantly by economically and socially deprived minority groups. Eighty-seven per cent of the narcotics cases lived in 271 of the 1,558 census tracts in Manhattan, Brooklyn and the Bronx—that is, in 17% of the tracts in these boroughs.

Census data were used to further investigate the characteristics of neighborhoods of high drug use. In this analysis, we considered *only* the 271 tracts where the great majority of narcotics cases lived. Among the neighborhood characteristics consistently related to high drug use in each of the three boroughs were poverty, lack of intact family living arrangements, crowded dwelling units, lower occupations, low television ownership and a high percentage of minority group members. The same general pattern emerged even when we limited our focus only to the census tracts within Harlem. Significantly higher drug use was found in tracts with low average education, lower occupations, crowded dwellings and low rental.

Heroin is no respecter of color or place of birth. It is our belief, based on the data we have collected, that the use of drugs may become epidemic in any area where there is a breakdown in the web of human relationships. Many native white boys use heroin, and in areas like Harlem, there are local neighborhoods in which no drug use was reported in the period of study. In even the "worst" neighborhoods, the vast majority of young people do not become heroin users.

In order to discover more precisely what it is in the epidemic neighborhoods that makes one boy succumb and not another, we are doing a number of additional studies on various factors such as the prevailing climate of thought among young teen-agers in these areas, family relationships, and the role of street gangs. We are also bringing our count of drug cases up to date to see if adolescent drug use is continuing at the same rate and if any new neighborhoods are becoming affected.

Weekly Business Statistics. LEONARD H. LEMPERT, *Statistical Indicator Associates.*

A check list of the major weekly business statistics includes about 70 Different series.

The problems associated with weekly statistics are manifold: weekly series represent somewhat different samples from corresponding monthly series; sufficient details are often lacking; weekly data frequently overlap two months; and most weekly series are not seasonally adjusted. In spite of such problems, a weekly composite index such as *The New York Times* Index of Business Activity moves in accordance with such business cycle landmarks as the reference dates of the National Bureau of Economic Research and with interim ups and downs such as those of the FRB Index of Industrial Production.

Judicious use of weekly business statistics enables the economist to obtain early estimates of the monthly figures of failure liabilities, unemployment, bank debits outside NYC, freight carloadings, FRB Index of Industrial Production, retail sales, and consumer installment credit. The reliability of the different estimates varies with the individual series and with the purpose for which the estimate is used. As a whole, these estimates provide, a "fair" to "good" picture of the previous month's overall business activity by the first or second week of the following month. As an auxiliary to the monthly and quarterly data available, weekly business statistics can play an important role in the economist's contemporary analysis of business activity.

Accelerated Life Testing of Capacitors. G. J. LEVENBACH, *Bell Telephone Laboratories, Inc.*

One of the problems meeting the designer of modern electronic communication systems is the ultimate reliability to be achieved under service conditions. The required level of reliability together with the unavoidable complexity of present day equipment call for components having extremely low failure rates. Evaluation of the rates of failure of such magnitude in the laboratory or pilot plant is practically impossible in a reasonable time and with a reasonable number of components. One way to meet this problem is the artificial reduction of the time to failure. This reduction of time or in other words the acceleration of life must be realistic to be of any value, which presents many problems in actual life testing. In recent years several papers have been published concerning statistical methods applied to life tests showing an exponential distribution of time to failure. These methods have been used as the basis for the design of an experiment to explore the acceleration obtainable in testing capacitors of recent design. Some results of this experiment estimating acceleration factors corresponding to increases in applied voltages and ambient temperatures are presented.

Effects of Automation on Occupational Structure. RICHARD H. LEWIS, *Dept. of Labor.*

Automation will have a significant effect upon the occupational structure of the labor force. However, for at least the next ten years, the changes in relative importance of different occupations will be moderate. There will be increasing requirements for professional, technical and skilled workers, but the semiskilled workers, laborers and routine clerical workers, who make up the bulk of the industrial labor force, will by no means be eliminated. The changes that will occur represent a continuation of the trends in occupational employment.

Automation has not yet been introduced to a wide extent, and where it has there is little information on its employment effects. For this reason, it is not possible to make a quantitative statement of the overall changes that automation has already brought. In looking toward the future, we can be guided by our knowledge of which industries offer the greatest opportunities for automation and how the occupational structure of these industries will be affected.

Automation will continue to make extensive inroads into industries, such as electronics, automobiles, machinery and chemicals and petroleum refining. In other industries such as lumber, textiles and apparel, which have many semiskilled workers or laborers, there is little possibility of substantial automation in the immediate future, although technical progress will continue to be made in these industries. The long term growth of employment in technical and skilled fields which will be reinforced by automation, has important implications for our educational systems and training programs.

The Role of Middleman Transactions in World Trade. ROBERT M. LICHTENBERG, *National Bureau of Economic Research*

In the course of the Study of the Structure of World Trade and Payments, under way at the National Bureau of Economic Research, it has been found that we need to know more about the role of middlemen in international trade. This paper examines the available data on trade involving purchases by final consuming countries from countries other than the country of origin, and it considers some of the economic and statistical aspects of this trade.

Seven countries, mainly Western European, report imports both by country of production and country of purchase. These records supplemented by qualitative and rougher quantitative material on other countries indicate that the bulk of world trade is direct. In 1952, middlemen supplied about 15 per cent of the imports of the seven countries. Middlemen mainly supplied more or less refined foods and materials to the seven countries; they reside principally in a few industrial countries and their outlying entrepot centers (e.g. Hong Kong). A major part of the exports of many Asiatic and African and some Western Hemisphere countries goes to final importing countries via middlemen and we have some evidence that imports of these countries also come via middlemen. German and Danish figures show Western European metropoles selling a high percentage of produce of their own overseas territories, but also that this is less than half of middleman trade. Most middleman business is done offshore. Re-exports and entrepot records of middleman countries cover only a small part of the trade.

We find that: (1) using trade and payments records to assess the interregional financial position of countries results in some serious distortions; (2) middlemen make a substantial contribution to the foreign exchange position of some countries both directly and through the support they lend to complementary services; (3) middlemen play a major role in linking the economies of many underdeveloped countries to the world economy; (4) middleman activities introduce considerable flexibility into bilateral and regional payments agreements by substituting commodity arbitrage for currency arbitrage, and they are a factor tending to push sterling and, hence, other currencies toward convertibility.

Some Sampling Problems in the Construction of Price Indexes. ROBERT E. LIPSEY *National Bureau of Economic Research.*

Most commonly used methods of sampling for price indexes, such as selection of the most important commodities or of those for which price data are readily available, are likely to bias the indexes. In particular, they tend to give too much weight to the behavior of crude materials and foods, at the expense of highly manufactured goods. Stratification of the price universe enables one not only to increase the precision of the index number but also, and more important, to reduce the bias resulting from the concentration of under-sampling and non-response in certain groups of prices. Published estimates of the sampling errors of price indexes are too low. Stratification would reduce the bias in these estimates due to the treatment of non-random samples as simple random samples (or the incorrect assumption that each dollar of trade has an equal probability of inclusion in the sample). Estimates of sampling error which treat a sample as a simple random sample will always be lower than those treating it as a sample stratified by probability of inclusion, although even the latter are only minimum estimates.

The Teaching of Statistics at the University Level: An International Survey. B. A. LIU, *United Nations.*

The Unesco survey of university teaching of social sciences in selected countries was extended to teaching of statistics, and reports were collected from 14 countries by the International Statistical Institute. This paper presents some extracts from the reports on the United States, United Kingdom, France, Sweden, India and Yugoslavia.

Among interesting items from these reports:

(a) *U.S.* Of 117 doctoral graduates in statistics during 1950–1953 from 25 institutions, almost half came from North Carolina, California, Columbia and Iowa State Universities.

(b) *U. K.* Of some 240 statistics courses given during 1954–1955, about one-third were in economics departments, one-fifth in mathematics and science, and the rest in psychology, statistics, agriculture, biology, medicine, sociology, etc.;

(c) *France.* Statistics, with applications to economics, now given in most law faculties; some science faculties offer probability theory and mathematical statistics; a few lectures on statistics are introduced in the medical faculties of Paris and Rennes; none whatever in faculties of letters;

(d) *Sweden.* Courses in statistical methods are generally obligatory for students of sociology, psychology and pedagogy;

(e) *India.* Increasing demand for admission into statistics courses severely taxes the existing staff and capacity in most universities;

(f) *Yugoslavia.* Over 6,000 students enrolled in statistics courses in 1953–1954, two-thirds in economics faculties, one-third in faculties of law and agriculture and departments of mathematics and psychology.

Using Census Tracts in Epidemiological Studies. ELEANOR J. MACDONALD, *University of Texas.*

The epidemiological approach was applied in a study of three outstanding disease problems facing Houston: tuberculosis, poliomyelitis and cancer. Information from the 119 census tracts was combined into 18 trade areas in each of the morbidity and mortality categories under consideration, by age, race, sex and color. Standardized rates and differences in each of these categories by trade area were figured and evaluated. All factors requiring consideration in reviewing the disease problems were obtained from the Chamber of Commerce by trade area. The use of census tract data makes possible more efficient distribution of public health services. Recommendations based on findings pinpointed to such small areas are applicable without the usual delay.

City Annexations and Their Effect on Tract Boundaries. WILBUR E. MCCANN, *Los Angeles Chamber of Commerce.*

Practitioners of the Census Tract system in California, and particularly in Los Angeles, have been plagued for some time with the vexing problem of city annexations. On the one hand, Tract boundaries should remain intact over

a period of time to insure comparability, and yet they must yield to the artificial changes in political boundaries occasioned by annexations, or at times, by the creation of an entirely new city.

The dilemma thus created is particularly serious because these artificial changes are more prevalent in the very areas that have the most urgent need for the type of analysis the tract system usually affords. In Los Angeles, an attempt has been made to solve this problem in part by the practice of establishing a new Census Tract for each new portion annexed. In so doing, however, the Tract system has become very complex and often "illogical" from the point of view of the user, particularly private business.

In this three-way tug-of-war between the desire for comparability, legal purity and practical usage, there is really no wholly satisfactory solution in an area that is beset by frequent and often indiscriminate annexations. As revolutionary as it may appear, I think the best solution is to ignore city boundaries and establish tracts solely on the basis of geographic, economic and social factors. Such drastic action need only be taken where annexations or illogical city boundaries are an acute problem. There are obvious objections to this plan, but it has a basic advantage that can not be overlooked. For the first time it would permit the establishment of scientifically sound tracts whose boundaries could be retained over a period of time without artificial interference. The Tracts along the perimeter of cities would make just as much sense as the Tracts in the interior parts of cities. The major disadvantage of this method might be mitigated by arranging the Enumeration Districts in such a way that detailed city data could be provided to those who must follow city boundaries.

Interlaboratory Studies. JOHN MANDEL, *National Bureau of Standards.*

Many interlaboratory studies are conducted for the purpose of establishing the merits of a method of measurement, generally in reference to a standard method. In the work reported in this paper, the method under investigation is the measurement of the smoothness of paper by means of the Sheffield Smoothness Tester, in reference to the method based on the Bekk Smoothness Tester. The study was designed in two steps: A single-laboratory study and an interlaboratory test, the latter not yet completed. In the single-laboratory study, conducted at the National Bureau of Standards, the merits of the two methods were compared by means of the sensitivity-criterion. The study involved the establishment of the relationship of the two methods, as well as the determination of the precision of each method and of the factors affecting precision. The results based on close to 6000 measurements apportioned among three operators are reported and it is pointed out that general conclusions can only be drawn after confirmation of the obtained results by an interlaboratory study. The opinion is expressed that in most cases, most of the desired information should be obtained in a well-designed test carried out in a single laboratory, the role of the interlaboratory test being confirmatory rather than exploratory.

Inventory Fluctuations and U.S. Imports. EDWARD MARCUS, *Brooklyn College.*

Business cycle fluctuations in the United States assume great importance for the economies of the other countries because of the magnitude of our foreign trade. These variations influence both economic activity abroad and the balance of payments positions of our trading partners. Hence the significance attached to the means and degree of cyclical transmission. The 1948–50 cycle in this country was predominantly one dominated by inventory movements. However, when we analyse imports we see that such was not the case—at least not for the major commodities. While changes in stocks held in this country affected the volume of imports, other factors also played important roles, especially relative prices and the marginal nature of foreign supplies. The competition between natural and synthetic rubber and Canadian and Swedish wood pulp are examples of the price influence; that between domestic and foreign wood pulp and domestic and foreign lumber illustrate the marginality possibility. Hence, it is possible for the foreign exporter to cushion, to some extent, the impact on him of our internal fluctuations. By proper price policies he can gain access to our expanding market and reduce the possibility of being ousted when demand declines. If he succeeds he will also contribute to lessening the dangers to his country's external balance, through stabilizing export volumes and thus dollar earnings.

The Impact of Taxes on Industry. WALTER MAYNARD, *New York Society of Security Analysts.*

The corporate income tax at its present high rates and under today's conditions is harmful in that it (a) functions as a capricious and variable sales tax, (b) provides an incentive for corporations to finance themselves with debt, (c) causes the government to bear the major share of corporate extravagance and (d) tends to promote "big business" to the detriment of small business.

High and punitive rates of personal income tax force business to organize in corporate form, accentuate the attractiveness of the pension plans and stock option plans which enable big business to lure able people from small business and partnerships, prevent the accumulation of venturesome capital, and thus tend to promote concentration of economic activity in the hands of big business.

The security analyst is in an exceptionally good position to observe the economic effects of the capital gains tax, which, despite the small revenues that it produces, are powerful and, in the main, harmful. The capital gains tax is not a true tax on capital gains, but rather a tax on the realization of capital gains. In other words, it is, in effect, a varying-rate transfer tax which prevents transfers when its effective rate is so high as to take a substantial part of the value of the property transferred.

What would be logical reforms for the grave economic faults in our present tax system? First, maximum income tax rates should be lowered to levels where they are no longer punitive and do not provide incentives for diversion of the nation's energies from productive channels to tax avoidance. This action, which would be a powerful economic stimulus, would probably not produce any lessening in tax yield. Second, the corporate income tax should be set at a level in line with the basic personal income tax rate. The revenues thus lost could readily be made up via excise or sales taxes. Third, we should recognize that the capital gains tax is a transfer tax at such a high rate that it inhibits the capital mobility which a growing economy needs, and cure this harmful effect by setting a reasonable limit on its impact. Such a step would undoubtedly increase the presently small revenues from this tax.

Industrial Productivity in Relation to the Cost of Management. SEYMOUR MELMAN, *Columbia University.*

Changes in methods of industrial management have been a part of the world-wide efforts to raise industrial productivity. It is commonly assumed, for example, that the management methods (and thereby costs) of American firms are necessarily linked with the levels of productivity attained in production operations. This paper tests the validity of such assumptions. Rates of change in productivity and administrative overhead are not necessarily related. Levels of industrial productivity at given times are not necessarily accompanied by related outlays for managing industrial firms. The absence of correlation stems from the fact that these two processes in industrial firms have been powered by substantially separate causal systems.

Industrial productivity in business-managed industrial firms has been a function of degree of mechanization which, in turn, has been largely governed by the relative cost of labor to machinery. The cost of management has been determined by the extension of the scope and intensity of managerial decision-making. These two dynamics have operated as separate processes in industrial firms, independently of the fact that the proximate decisions in each case have been made by the same group of occupations, the mangers and their staffs.

Social Welfare Expenditures in the United States—Problems of Measurement and Analysis. IDA C. MERRIAM and DOROTHY MCCAMMAN, *Social Security Administration.*

Measurements of social welfare expenditures assume importance when the aggregate is significant in relation to national output or when competing claims for economic and social purposes must be weighed.

A basic problem of measurement is determining what to include as a social welfare expenditure. The suggested definition includes only expenditures under programs directly focusing on individual welfare—education, public health, social security—and excludes expenditurees under such programs as TVA which, although contributing to social welfare, are economically oriented.

Assuming this basic definition is valid, there remain questions regarding inclusion or exclusion of specific programs and of classification. These include: Are veterans' payments welfare or war-related expenditures? Should medial benefits under public assistance, workmen's compensation, or veterans' programs be classified with each program or lumped as medical services? What treatment is proper for programs like workmen's compensation where benefits required by law are commonly provided through private insurance?

The Effect of Closing Rate on Visual Detection Probabilities. ROBERT MISEKY, *Cornell Aeronautical Laboratory, Inc.*

If $P_1(r)$ is the cumulative probability of detection of a moving object at range r when the observer and object are closing at a speed v_1, then it is shown that the cumulative probability of detection $P_2(r)$ for a closing speed v_2 may be found from the relation

$$Q_2 = Q_1^{v_2/v_1}$$

where $Q_1 = 1 - P_1$ and $Q_2 = 1 - P_2$.

The result is based on the proof that for large n and m

$$\left(\prod_{i=1}^{n} q_i\right)^{1/n} \text{ is approximately equal to } \left(\prod_{i=1}^{m} q_i\right)^{1/m}$$

where q_i is the probability of no detection in a small interval.

A Program of Studies of Scientific Research and Development. JAMES M. MITCHELL, *National Science Foundation.*

One of the major responsibilities of the National Science Foundation (established by Congress in 1950) is the development and encouragement of a national policy for promoting basic research and education in the sciences. This is being done, in part, with the aid of information gained from a group of inter-related surveys of such matters as the sources of funds for scientific research and development, the expenditure of these funds, fields of research covered, and type of research scientist and engineer personnel employed by various groups in industry, the universities, and the Federal Government. As part of this undertaking, the Bureau of Labor Statistics, under contract, surveyed the research and development activities of private industrial firms.

These inquiries into the dimensions of the nation's scientific research and development have encountered many difficulties of concept and definition, of response, of interpretation, and of comparability. Yet they are extending our knowledge of the varying goals of the institutions in the research community, of patterns of organization for research, of some major financial and institutional inter-relationships and problems arising from them, and of the likely effects of possible alternative Federal research policies.

On Improving the Timeliness of Current Economic Statistics. CAREY P. MODLIN, JR., *Office of Statistical Standards.*

The plea for greater speed in the publication of current economic statistics is a rational assignment of high priority to an important need felt by persons to whom current economic statistics are essential. It is a plea that current economic statistics be available in time to permit anticipations of economic developments. Timely statistics can be achieved through: (1) earlier final estimates, (2) preliminary estimates, or (3) reduction in the period covered and more frequent publication. Substantial costs of various sorts have to be met to produce data earlier; these costs take the form of additional reporting burden on respondents, additional operating costs for the collecting agency and somewhat less accurate reports and estimates. An "early estimates" bias is peculiarly important in preliminary estimates for anticipatory current economic statistics. The choice between the three methods of achieving greater timeliness depends upon the needs of the users, the resources available, the money and quality costs of the speed up and, to some extent independently, upon the nature of the activity measured. Recent progress in speeding up the availability of specific series and the needs for additional progress in those series were reviewed, and the likelihood of future progress was appraised.

Recent Developments in Business Financial Statistics. CAREY P. MODLIN JR., *Office of Statistical Standards.*

The series in business financial statistics are assumed to be those covering sales, inventories, corporate profits, income statements, balance sheets, expenditures for new plant and equipment, statements of sources and uses of funds and related items. Business financial statistics have been assigned a more important role as current economic indicators by various groups, including the Congress and both government and private users of Federal statistics. With the larger role has come a more searching critical appraisal of the ability of our existing business financial statistics to play the larger role. The result is that the most important recent development in this area of statistics is the recognition and publicity given the need for improvement in the existing series. Many improvements needed in these series were suggested in recent reports to both the Congress and to Executive Branch agencies. Specific improvements already accomplished in series on expenditures for new plant and equipment, quarterly financial reports, sales and inventories, pension funds, annual financial reports, enterprise-establishment relations, and money-flows are noted.

Effect of the Recent Recession and Recovery on the Factory Workweek. SAM A. MORGENSTEIN and MARTIN L. MARIMONT, *Bureau of Labor Statistics.*

The Bureau of Labor Statistics has recently retabulated its employment and man-hour sample for the months of May 1953, 1954, and 1955 to obtain distributions of production workers' average weekly hours for reporting factory establishments. The month of May in these years fell successively during the periods when industrial activity was at its peak, had dipped into a recession and finally was well on its way to recovery. A review of these data therefore provided information concerning the relationship of changes in the business cycle to changes in hours worked.

Some of the major findings are:

a. For the period 1953–1954 the major shifts occurred in two of the categories: the percentage of workers in the over 38 but under 42 hours category rose from 36 to 43; the number of workers in the 42 hours and over category declined from 40 per cent to 24 per cent. b. From May 1954 to May 1955 employment in the 42 hours and over group had risen to 38 per cent—and fallen in the 36–38 hours group from 34 to 13 per cent. c. The distributions for the pre- and post-recession months are somewhat similar with some exceptions: in May 1955 there were more employees in the 39–41 hour classes and fewer in the 42 and over group than in May 1953. Thus, while overtime was worked in 1955 by the same proportion of workers it was of shorter duration. d. On a geographical basis the Western States showed little change in the distributions while the Northwestern and North Central States showed an exaggerated version of the national change.

Despite the business decline about 75 of every 100 production workers in 1954 were in factories averaging better than 38 hours per week—although in 1953 this was true for 86 of every 100 workers. In the May 1955 recovery period, workers in establishments averaging more than 38 hours per week had risen to 87 per cent of the total. The data on which this study is based are those which are regularly collected under the Bureau of Labor Statistics—State Co-operative Program for the monthly collection and preparation of employment, earnings, and hours of work. Data are collected for the pay period ending nearest the 15th of the month and production-worker man-hours represent all hours worked or paid for during the specific pay period. Over 44,000 manufacturing establishments covering approximately 65 per cent of total manufacturing employment volutarily submit monthly reports. These are classified industrially using the Standard Industrial Classification Manual of the U.S. Bureau of the Budget.

Monthly Production Indexes and Changes in Output per Manhour. MILTON MOSS.

Comparison between the Federal Reserve index of industrial production and manhours indicates that output per manhour in manufacturing rose 40 per cent from early 1947 to late 1955, rising more than 4 per cent per year for most of the period, but less when output slackened. The two areas of the industrial production index, one based monthly on adjusted manhour data and the other on physical product figures show strikingly similar long-term changes in output per manhour for most of this period. Monthly changes in output per manhour for the adjusted manhour area of the index are interpolated smoothly between levels of independently derived annual indexes based on comprehensive output information. For the aggregate of monthly series based on adjusted manhour data, annual averages of montly indexes are practically identical to a corresponding composite of independent annual indexes. These levels are subject to further review based on benchmark indexes for 1954 relative to 1947 being constructed by the Census Bureau and Federal Reserve. In order to develop improved short-run estimates of output per manhour for the manhour component of the production index for current periods the Board is experimenting with the use of indicated changes in output per manhour in the monthly physical quantity component of the production index.

Experimental Design in Flame Photometry Determinations. BARRY W. MULLIGAN and ALAN F. HAUGHT, *National Bureau of Standards.*

In developing a method for determining the alkalies in certain refractory materials, it was observed that the readings of the analyzing instrument, a flame photometer, were shifting with time. In order to measure the magnitude of the shift and to compensate for it, the statistical method for measuring instrumental drift published by W. J. Youden (*Science*, Oct. 22, 1954) was applied. The statistical method improved the precision and provided a measure of the drift, except when quantities of widely different magnitudes were intercompared. Subsequent investigation, again utilising the statistical method of incomplete blocks, showed that the drift resulted from fatigue of the photo-electric tube involved in the measurement, and that the drift was dependent upon the intensity of the light to which the tube was exposed. This variation of the drift with the magnitude of the measurement was contrary to one of the assumptions of the statistical method. To overcome this defect the procedure was modified so that quantities were grouped according to order of magnitude before final measurements were made and the drift correction applied. Improved experimental design in this investigation resulted in accurate measurements being obtained despite appreciable drift in the instrument.

The Statistician's Role in Allocating National Budget Expenditures. CANDIDO OLIVERAS, *Puerto Rico Planning Board.*

The role of the statistician in allocating national budget expenditures is not limited to assisting in the development of the immediate decisions with regard to the allocation of funds to specific programs and projects. Instead, the statistician must collect the statistics required for the determination of economic and social goals in quantitative terms in light of the objectives or desires of the persons in political power. In the case of Puerto Rico these objectives are to raise the levels of living. The statistics generated must enable the translation of these quantitative goals into cost and benefit answers so that the decision makers may be in a position to make decisions regarding alternative uses of limited fund resources. Since the allocation of expenditures is a continual process, occuring each year, there is need for periodic reviews which should bring to the decision-maker statistics that are newly generated from the operation of a program. By means of reviewing the Puerto Rican system for allocating budget expenses involving the preparation of three formal documents (The Economic Report to the Governor, the Six-Year Financial Program and Model Budget), the authors pointed to the need for: (1) many general statistical series such as national income accounts, labor force surveys; (2) many special statistics such as electric power consumption by various strata, and operating data on specific program; (3) analytical investigations such as input-output studies; and (4) economic and social models of the future. At all times the statistician must understand that the final decision with regard to the allocation of budget expenditures must rest with the politically accountable.

New Findings on Upper Income Shares. VICTOR PERLO, *New York City.*

This study finds no significant decline between 1929 and 1948 in upper income group shares in the national income. Other studies which purport to show sharp declines in these shares fail to allow for new forms of non-reported income, and increased use of legal devices for non-reporting of income, motivated by the sharply higher tax rates of the World War II and subsequent periods. Here upper income shares are calculated for 1929, 1939, 1945, and 1948 as follows: 1. Tax-reported income is taken for the top groups, identified in relation to the total number of income receivers, with no adjustment for income splitting. 2. The difference between tax return income and total income (Commerce) for each type of property income is allocated proportionately at all income levels. Additions are made for tax-free supplements to executive salaries. 3. Undistributed corporate income, interpreted as effective capital gains, is attributed to stockholders in proportion to dividend receipts. The results show the share of the top 1% steady at round one-sixth, and of the top 5% at around one-third, of adjusted national income throughout the period.

Applicability of Distribution-Free and Other Simplified Statistics in Medical Research and Biometry. RICHARD D. REMINGTON, *University of Oregon.*

To ascertain applicability of certain new "quick and easy" techniques in problems of medical research, 629 recent publications of the staff of a university medical school are reviewed for content. Thirty per cent are classified as "experimental" reports, and these are analyzed in detail. Statistical problems are found implicit in nearly all the studies of this group, but statistical techniques are used less frequently. Most common problem is that of judging a difference between two samples. Although classical methods are still not fully exploited, several non-parametric and simplified methods can be recommended as convenient and sufficiently powerful to be used with conviction on typical problems in medical research when speed or simplicity of analysis is required, or when the usual parametric assumptions do not obtain. The sign, run, rank sum, multiple range and other simplified tests are described and discussed. An annotated bibliography is included.

The Development of the Flow of Institutional Savings in the Analysis of Treasury Borrowing Problems. R. DUANE SAUNDERS, *Treasury Dept.*

In the analysis of Treasury borrowing problems during World War II, the Treasury Department staff developed new techniques in studying the many different markets for Government securities. Essentially these analyses explored the sources and uses of funds by various economic sectors. The Federal Reserve's new "Flow of Funds System of National Accounts" is a much more comprehensive version of this sources and uses technique. In the Treasury studies the primary emphasis was on liquid savings and the flow of funds into financial institutions. During the War Loan drives, with the exception of the First War Loan, these analyses were found useful by policy officials in establishing war loan goals. During the drives and in the planning stages of the drives the sources of funds analyses were used to show the derivation of the sales goals for each investor class, and more important, to prove to the hugh volunteer sales force that the money was there. Since the war the Treasury staff has continued to use the sources of funds as an essential part of the background for financing operations. However, the shift in economic environment from war to peace has made it necessary to devote more study to the supply of investments other than Federal securities—which had a virtual monopoly in World War II. In the meantime our knowledge of institutional behavior has been much improved by better and more comprehensive statistics on savings and also on the supply of investments. As institutional patterns of behavior change rather slowly, changes in direction are frequently evident before they become pronounced. The flow of funds technique is also helpful in that it provides a set of internally consistent results. However, these results are no better than the analyst's original set of assumptions.

Construction Statistics. WALTER W. SCHNEIDER, *Department of Commerce.*

Construction is the largest industry in our country, approaching $60 billion; directly and indirectly it provides employment for 10 million persons and is the largest consumer of strategic materials. Construction statistics are vitally important in public policy decisions and in the millions of decisions of private industry in promoting high levels of economic activity. In spite of their importance, present construction statistics have major deficiencies which according to the U. S. Chamber of Commerce makes them a threat to the formulation of sound public policy. Authorities say construction statistics are one of our nation's major statistical problems. Major shortcomings exist in current measures of new construction activity, housing starts, and employment estimates. No current reliable figures are

available on the volume of alterations and repairs, materials use, the trend of construction prices, and the housing inventory. Minor improvements are being made but most major deficiencies cannot be overcome with the present limited resources. Several programs are now being considered for presentation in the 1957 Budget Estimates. These programs would provide a sample national housing inventory; development of reliable estimates of new construction for State and Local Governments and public utilities, and of alterations and repairs to all types of structures; for making studies of materials consumption and labor requirements; for appraising factors affecting housing demands and for technical research leading to other basic improvements.

The Comparison of the Sensitivities of Experiments Using Different Scales of Measurement (Preliminary Report) DANIEL E. W. SCHUMMANN, *Virginia Polytechnic Institute.*

This paper deals with a problem formulated by W. G. Cochran in a paper in the Annals of Mathematical Statistics (Sept. 1943) on: "The Comparison of Different Scales of Measurement for Experimental Results." Assume two experiments (equal in size and design) with identical treatments, both depending on the analysis of variance and the scoring scales used. If real treatment differences exist, the more sensitive experiment or scoring scale will obviously be the one that best shows up the treatment differences, and a comparison of the sensitivities should be based on the variance ratios of the two experiments. Hence, in comparing two scales for more than two experimental treatments, we need a test of significance that two noncentral variance ratios (noncentral F's) are equal. Considering the case of two noncentral F's, both with $2a$ and $2b$ degrees of freedom and with parameters of noncentrality λ_1 and λ_2 (where $\lambda = \Sigma t_i^2/2\sigma^2$, t_i being the ith treatment effect and σ^2 the error variance, in the usual additive model of analysis of variance), the ditribution function of the ratio of two noncentral F's is obtained. Some properties of this function and methods of finding the probability integral, $\int_0^{w_0} f(w)dw$ are considered. The construction of tables of percentage points for use in tests of significance under the hypothesis that $\lambda_1 = \lambda_2$, i.e. the hypothesis of equal sensitivity of the two experiments is dealt with.

Capital Costs of Urbanization. CAROLYN SHILLING, *Columbia University.*

This paper provides quantitative support from Soviet data for the proposition that, where industrial growth must rely primarily on the recruitment of labor from rural areas, investment in urban housing and community facilities needed for this transfer of population is sufficiently high to warrant the alternative of concentration on labor-saving production processes including, at the extreme, automation. The Soviet Union has been chosen as a testing ground because with nearly fifty per cent of the population in rural areas the choice of urbanization remains a real one, unemployment and even under-employment of labor has virtually disappeared, and the government is in fact concentrating on means to raise productivity in the non-agricultural sectors of the economy. Finally, Soviet experience in semi-automatic and automatic production has been greatly augmented in the post-war period and even approaches in some lines, such as the production of automobiles, technological levels found in the United States.

The estimated costs of urbanising a single Russian peasant include the net costs of housing, some public utilities, schools, hospitals, retail outlets, etc., not only for the in-migrant and his dependents, but also for secondary workers required to service them in the urban environment. Refinements, such as costs of training and compensating investment in agriculture, are also taken into account. The alternative costs of automation per replaced worker have been computed over a wide range of industries, but with particular emphasis on machine-building. These rough computations indicate in all cases that automation is less capital-using than urbanization and, depending on the industrial branch and level of mechanization, the savings reach several hundred per cent.

Seasonal Adjustments by Electronic Computer Methods. JULIUS SHISKIN and HARRY EISENPRESS, *Bureau of the Census.*

More than 1,200 time series have been adjusted for seasonal variations at the Bureau of the Census on our electronic computers during the past year. The method used for this purpose—Method I—is an elaboration of the standard ratio-to-moving average method for moving seasonal adjustments. A number of weaknesses in Method I have recently been corrected in a new method. In Method II the trend-cycle curve is traced out by a weighted (Spencer) 15-term moving average, which is smoother than the 5-month moving average of Method I and just as flexible. Smoother curves are also fitted to the seasonal-irregular ratios to provide seasonal adjustment factors. Extreme values among the ratios are isolated by a system of control charts and are replaced by averages of the extreme ratio and surrounding ratios. A 3-month of a 3-month moving average is then fitted to the seasonal-irregular ratios. Also, the curves fitted to the seasonal-irregular ratios in Method II follow the ratios for the full period of the data. Finally, Method II has been programmed for series of from 6 to 30 years. Studies of test series indicate that these changes improve seasonal adjustments. A regression method of seasonal adjustment which takes into account changes in the seasonal factors at different levels of activity as well as changes over time has also been programmed and the results are now being tested.

Method II also yields for each series measures of the average monthly amplitude of the trend-cycle, the irregular, and the seasonal components of the series. Such measures computed for 34 test series indicate that during the post World War II period, month-to-month cyclical movements have been small compared to seasonal and irregular movements. We believe that these new seasonal programs are bringing us fairly close to a mechanical method of providing on a mass basis seasonal adjustments of the quality previously obtained only for a small number of series by a combination of laborious hand methods and professional judgments.

What Concepts Are Appropriate to Consumer Price Indexes? IRVING H. SIEGEL, *Twentieth Century Fund.*

This paper asks for (1) more and better consumer price measures, (2) greater awareness of the limitations of available measures, and (3) greater interest in the magnitude and direction of the difference between preferred indexes and constructible ones. Many meaningful consumer price indexes could be made if ideal data were available. None could be labeled as "best" without reference to a context or use, but a close specification of purpose would rule

out many otherwise eligible measures. In any case, consumer price measures, like all other index numbers, have arbitrary elements and cannot be rationalized in terms of the traditional theory of economic choice. Although many consumer price indexes may be defined in principle, few are actually available. The best known comprehensive measures, prepared for urban-worker families by the Department of Labor and for farm families of the Department of Agriculture, are used in many contexts for which they are not conceptually appropriate. Many additional price indexes are desirable—for example, for workers in additional cities, for persons in higher income groups, and for farmers raising different major crops. Still other index number needs should be noted. One is a suitable measure of retail prices for deflating national consumer expenditures. Another is the "fixed composition" index, an alternative to the conventional chain index for periods of significant change in the assortment or quality of commodities and services. A third need is a measure reflecting mortgage and installment financing in the monthly pricing of homes, automobiles, and other consumer durable goods.

Survey of Industrial Research and Development. WALT R. SIMMONS and FRANCES M. WATTENBERG, *Bureau of Labor Statistics.*

The survey of industrial research and development, conducted in 1955 by the Bureau of Labor Statistics under contract with the National Science Foundation, covered all commercial and industrial companies in the United States, excluding farms, governmental, educational, and non-profit organizations, and those research laboratories and consultants which serve the general public.

The sampling frame was constructed principally from companies liable for taxes for Old Age and Survivors Insurance. The over-all sample was a probability design with optimum allocation of resources to the initial selection of 11,600 companies from 225 strata. Stratification and allocation took into consideration industry, size-of-company, research intensity, expected response rate, and expected costs. Estimation included adjustment to employment controls from the frame by usual techniques for ratio estimates. The ratio estimates were chosen as much for their value as controls over non-sampling operations as for their utility in lowering variances. Three mail follow-ups to the original mailing culminated in an 89 per cent response to the survey. A subsample was also selected and contacted by telephone for use in checking for possible bias of non-response. Results of these tests indicated no statistical bias.

Because of the great interest in the subject of Industrial Research and Development, future surveys of this field probably will be made. Experience and knowledge gained from this survey will simplify the planning stages of the next survey.

Measurement of Industrial and Commercial Employment in the United States. WALT R. SIMMONS, *Bureau of Labor Statistics.*

The extensive range, scope, and variety of industrial and commercial activity in the U.S., are described most frequently through the common denominator of numbers of workers employed, classified into appropriate categories. For the current detailed picture by industry and geography the nation relies principally on monthly reports produced by the cooperative Federal-State BLS 790 Program. Technical aspects of that program are described. Attention is given to Basic Objectives, Administration and Program Organization, Leading Principles and Statistical Standards, Estimating Patterns, Benchmarks, Sample Design, Editing and Computation, and Precision of Results. Several underlying themes of this program appear at more than one place in the report. One of these is the size of the operation, which yields more than 13,000 separate estimates each month. Another is the emphasis upon continuity, which is a watchword of the system, and which influences many decisions on procedure.

The paper explains the BLS Link and Taper estimating procedure, and describes efforts to measure overall error of the estimate, including discrepancies from both sampling and measurement.

Variance Components, Finite Populations and Experimental Inference. H. FAIRFIELD SMITH, *North Carolina State College.*

Variance components computed as for the random model bear the same relation to second moments of complex finite universes as do k_2-statistics to simple finite universes. For any given experiment they form a definitive set of statistics free from dubious postulates. The effect of postulating particular universe sizes can be considered when making inferences. By introducing these postulates only at such later stage we avoid an undesirable rigidity which is often imposed when they are made part of the initial model. Just such rigidity seems responsible for much debate about the mixed model consequent upon changing viewpoints to which initial restrictions may not be applicable. It is therefore proposed that statistics computed as if the random model were being used be accepted as a standardized definition of dispersion components for any experiment or complex universe. Such acceptance may simplify usage and encourage flexibility of thought about appropriate inferences.

Applications of Operations Research. ROBERT H. SMITH, *Stanford Research Institute.*

Operations research can be defined as "the scientific approach to evaluating the alternatives under management control." Three industrial operations research studies which illustrate this definition were described. The purpose of each of these studies was to narrow the range within which management must rely upon intuition by measuring the results of alternative courses of action. The first application of operations research which was described was a study of the purchases, sales and inventories of a gas utility company. The study first described graphically the way in which the system operated. It then presented a system of equations which described the system mathematically. By the use of these equations, the management could determine the results of alternative operating policies with respect to the purchase, storage and sales of gas. The second application which was described was a study of the problem of controlling the inventory of a large railroad. In this case the operating rules which resulted were applied to the previous year's operations through the use of an electronic computer. The results under actual conditions of both the new and the old rules could then be compared. The third application which was described was the use of a matrix to determine the effects of alternative office space layouts in a large office building.

Oils and Light Metals. JOHN SPURDLE, *Dominick and Dominick.*

The domestic oil industry has made a handsome recovery from its status of the summer of 1954.

The foreign oil situation has been most satisfactory, both in regard to consumption gains and the orderly way in which Iranian and other new productions have been absorbed in a competitive market. Continuation of this pattern is expected, restraining the constant fear of domestic producers over dumping of foreign crude or products. A strong market for residual oil, increasingly assigned to foreign suppliers, has been an assisting factor in this general situation. Since the beginning of 1955 oil stocks in the average have improved much less than the market in general. To date the Standard & Poor's Industrial Average of 423 stocks is up 23%. Some of the oils have done better than others, but the majority compared unfavorably with this standard. The best of the group were the international companies, which did well. This was justified both by the operating results and the improving investor appraisal of foreign business operations. I am confident that there should be a substantial change for the better in investor appraisal of the oil stocks over the next six months.

The Parity Index and the Farm Expenditure Survey. B. R. STAUBER, *Agricultural Marketing Service.*

The Index of Prices, Paid by Farmers for Commodities and Services, including Interest, Taxes, and Farm Wage Rates—also known by the shorter name, Parity Index—is used in conjuction with the Index of Prices Received by Farmers for computing Parity Prices of farm products. Although the living goods component of this Index serves approximately the same purposes with respect to farm people as the Consumer Price Index serves for middle income urban wage and clerical workers, the most important considerations regarding its construction are its relation and similarity of construction to the Index of Prices Received by Farmers. A survey of 10,000 farmers is being conducted in February and March of 1956 to obtain detailed information concerning the expenditures of farmers for the various commodities used for living and production purposes in 1955. This information is to be used to modernize the weighting pattern of the Parity Index and to strengthen the estimates of net income from farming. The survey is being conducted through personal enumeration of respondents selected by a two-stage probability sample design from farmers reported by the 1954 Census of Agriculture. Perhaps the most difficult problem in current price collection programs lies in securing accurate information concerning the actual prices of administratively priced commodities such as automobiles and farm machinery. Particularly in times when, as during the war, premiums or bonuses were exacted from customers as a condition of sale, and in times such as the present, when large discounts from list prices or exorbitant trade-in allowances for used equipment are commonplace, dealers are loathe to report actual prices, and tend to quote list prices, thus underestimating the actual price increase in times of sellers' markets, and the decline in times of buyers' markets. Statisticians are urged to use their influence to promote accurate reporting in the interest of more adequate analyses of the economic situation.

Application of Statistics to Tire Testing. ROBERT D. STIEHLER, *National Bureau of Standards.*

The quality of tires purchased by the Federal Government is of interest because of the large dollar volume and the variability of commercial tires. Many tires are tested commercially on taxis, truck fleets, or other service vehicles. The poor precision of such tests justifies the added expense of controlled test fleets. The application of statistical design to these tests permits compensation for the effects of both systematic and random variables. Some of the designs that have been useful in the determination of treadwear are: latin and graeco-latin squares, and various types of Chain block designs. For example, one test based on a graeco-latin square design involved the comparison of 28 tire constructions and provided information on the effects of wheel positions, test periods, seasonal variations and aging of the tires in addition to the effects of the tire constructions themselves. In another test, 32 commercial tires were compared on the 16 positions of 4 test vehicles by means of a Chain block design with two-way elimination of error. Tests involving two, three, or four different tread compositions on the same tire carcass are also discussed. In all tire tests the proper statistical treatment of the data is important. The multiplicative nature of the factors influencing treadwear and the essential constancy of the coefficient of variation necessitate a logarithmic transformation of the data prior to analysis. In comparing various methods of test, the sensitivity criterion is useful, which considers for each method both its precision and the magnitude of the difference in measurements obtained on dissimilar tires. Despite the numerous factors affecting treadwear, reliable inferences can be obtained through statistical design and analysis.

Mathematical Models for Empirical Studies of Decision Making. PATRICK SUPPES, *Center for Advanced Study in the Behavioral Sciences, Stanford.*

This paper is a survey of mathematical models which are applicable in four major areas of decision making. (1) *Static individual decisions.* This area has been most thoroughly investigated both experimentally and theoretically. The basic models derive from the work of Ramsey and von Neumann and Morgenstern. A linear porgramming model applicable to the measurement of utility and subjective probability is discussed. (2) *Dynamic individual decisions.* The discussion here centers on relations between (i) the work of Estes, Bush, and Mosteller, and (ii) the ideas of game theory and multi-stage statistical decision theory. Connections between a dynamic theory of utility and a learning theory for differentiable rewards are also discussed. (3) *Static group decisions.* The fundamental problem for this area seems to be the construction of a model for interpersonal comparison of utilities. An obvious linear model is discussed. (4) *Dynamic group decisions.* Some applications of Estes' learning model are discussed, and empirical predictions compared with corresponding game-theoretic predictions.

Preliminary Results of the 1954 Census of Agriculture. CONRAD TAEUBER, *Bureau of the Census.*

Preliminary results of the 1954 Census of Agriculture indicate a continuation of long-time trends—a decline in the number of farms, an increase in the average size of farm, and increased use of mechanization and modern technology. The census found 4.8 million farms; a decrease of 11 per cent during the five years. Nearly half the decrease was in places of less than 50 acres and three-fourths related to farms with a value of sales of less than $1200. There was

no decline in the amount of farm land and the average size of farm increased from 215 to 242 acres. The decrease in the number of farms was at approximately the same rate for owners and tenants, although there was a decline of 74,000 in the number of sharecroppers. The number and proportion of farms with tractors, trucks, electricity, telephones, home freezers, and running water increased. Thirty-eight per cent of all farms had a tractor and no horses or mules, and 20 per cent had both a tractor and at least one horse or mule. The total number of horses dropped to 4.2 million. Total acres irrigated increased by nearly one-sixth. Increases were particularly marked in the Eastern States. Approximately two-thirds of all farms were classified as commercial farms in 1954, as in 1950.

Anticipations for 1956. ALAN H. TEMPLE, *The First National City Bank of New York.*

The country is closing its busiest and most prosperous year with business activity at the highest point of the year. Although aggregate production and trade have been expanding for nearly a year and a half, the latest aggregative figures are still trending upward. The rate of increase during much of 1955 has been between two and three times the long-term average growth. We cannot expect this rate to continue and the expansion phase may top off in the fairly near future. But so far as we know, it has not topped off yet.

The rate of expansion must slow down in part because there is no longer sufficient slack available in factory capacity, materials supply, and labor supply to support continuation of the 1955 rate of increase. Moreover, no reasonable person expects that all of the industries which contributed so strongly to the 1955 rise can supply a fresh stimulus in 1956. The contribution that they will make in 1956 is fully reflected, to say the least, in our present activity. Largely for these reasons, the recovery may reach its crest in the relatively near future. Certain signs should be expected to denote the crest. One would be a turndown in unfilled orders; second, cessation of inventory accumulation; and, third, end of expansion of installment credit. But unfilled orders are still rising; inventory accumulation is likely to go further before the trend reverses; and the lines of new installment loans and of repayments on outstanding loans have not yet crossed.

Assuming that a peak is in the offing, it is highly improbable that it will take the shape of an inverted V. Only in case inventory accumulation, speculation, and inflationary money or price-wage developments should run wild in the next few months would this be expected. The prospect rather is for a long crest or inverted saucer shape. While automobiles and housing may show some decline, they should still have a good year. And if their contribution to gross product is less than in 1955, there will be important offsets in other types of construction and public works and increased business expenditures for plant and equipment.

A slow rise to a moderately higher top and a probable sideways movement or slow decline after the top could and probably will carry so far through 1956 as to make the year on the whole a slightly bigger year than 1955. We should consider this a more satisfactory outcome than a continuation of the 1955 rate of increase. The economy is not free from strains, notably the expansion of debt at a rather rapid rate. The contry is enjoying full employment and general prosperity. The strains are not yet producing cracks. The probabilities for 1956 should keep us optimistic. The dangers should keep us conservative.

Comparison of the BLS Consumer Price Index and the AMS Index of Prices Paid by Farmers. LAZARE TEPER, *International Ladies' Garment Workers' Union.*

The index prepared by the Agricultural Marketing Service to measure prices paid by farmers for family living is based on out-dated, pre-war consumption patterns and contains numerous other deficiencies. The present AMS is full of faults despite its importance in evaluating the changes in the welfare of farmers. The bulk of information on which the index is based is gathered by mail; a critically large number of stores queried fail to answer. This is the case, for example, with about 70 per cent of food retailers and about 60 per cent of clothing stores.

The use of mail questionnaires prevents the use of proper specifications to insure the pricing of comparable items. The AMS index reflects, therefore, not only the changes in prices of comparable goods but also the effect of shifts in farmer purchases of goods of different quality. As a result the index is vitiated as a proper measure for determining farmers' welfare.

By comparison the consumers' price index, compiled by the Bureau of Labor Statistics to measure the average changes in prices paid by workers, seeks to measure changes in prices of goods and services of comparable quality. To insure comparability, the Bureau relies on specifications of goods and services it prices and relies primarily on personal visits for the collection of the needed information. Specially trained enumerators are used for this task. BLS index is computed directly on the basis of the information gathered in the field. On the other hand, the shortcomings of its data force AMS, in compiling average prices paid by farmers, to use estimates made by its staff rather than the price quotations gathered from respondents. This introduces a distinct subjectivity into the index.

The existence of disparate approaches to consumer price measurement unavoidably invites a certain amount of confusion among the users of the data. There is no reason that two different standards and philosophies should guide the measurement of living cost changes for farmers and workers. The matter deserves further study, both by governmental agencies and by students outside government.

The Derivation and Use of an Alternative to the Analysis of Variance. M. C. K. TWEEDIE, *Virginia Polytechnic Institute.*

Suppose that a fixed number n_i of independent observations x_{ij}, $j=1$ to n_i are taken on a variate x which has probability density $\exp(-\frac{1}{2}\mu^{-2}\lambda x+\mu^{-1}-\frac{1}{2}\lambda x^{-1})\lambda^{1/2}(2\pi x^3)^{-1/2}$, $0<x<\infty$. The maximum likelihood estimates of μ and λ are given by

$$\hat{\mu}_i = x_{i\cdot} = n_i^{-1}\sum_{j=1}^{n_i}(x_{ij}), \qquad \hat{\lambda}_i^{-1} = n_i^{-1}\sum_{j=1}^{n_i}(x_{ij}^{-1}-x_{i\cdot}^{-1}).$$

The conditional distribution of $\hat{\lambda}_i^{-1}$, at fixed $x_{i\cdot}$, is as $\chi^2_{n_i-1}/\lambda n_i$ where $\chi^2_{n_i-1}$ is a chi-square with n_i-1 d.f. This is

independent of both x_i and μ. Moreover, the distribution of $x_{i.}$ is the same as that of x except that λ is replaced by χn_i. If $i=1$ to N,

$$n_{\cdot} = \sum_{i=1}^{N} (n_i), \qquad x_{\cdot\cdot} = n_{\cdot}^{-1} \sum_{i=1}^{N} \sum_{j=1}^{n_i} (x_{ij}), \qquad \sum_{i=1}^{N} \sum_{j=1}^{n_i} (x_{ij}^{-1} - x_{\cdot\cdot}^{-1}) = \sum_{i=1}^{N} \sum_{j=1}^{n_i} (x_{ij}^{-1} - x_{i\cdot}^{-1}) + \sum_{i=1}^{N} n_i (x_{i\cdot}^{-1} - x_{\cdot\cdot}^{-1}).$$

The two sums on the right of this last equation are thus both distributed as $\lambda^{-1}\chi^2$, the first with $n_{\cdot} - N$ d.f. and and the second with $N-1$ d.f. The distributions are independent and the sum on the left is distributed as χ^2 with $n_{\cdot} - 1$ d.f. This provides an analogue of the analysis of variance, available for nested classifications, for testing differences between the N classes of i. The original distribution applies to first-passage times in a certain Brownian movement problem, and as an approximation elsewhere. Existing tables of χ^2 and F can be used for significance tests, etc.

Social Expenditures in Economic Development. N. T. WANG, *United Nations.*

Social expenditures are now recognized as an integral part of developmental programs. They frequently account for a fifth to a third of total development budgets. To gain a comprehensive view of the role of social expenditures in the national economy, developmental as well as non-developmental social expenditures, such as current expenditures and transfer payments, should be considered together.

The relative importance of social expenditures has grown over the last half-century with economic development. Social expenditures have reached 10–15% of gross national product in a number of developed countries today as compared with less than 2% around 1900. While many social programs may have been initiated by radicals, they have been consolidated or expanded by conservatives. (In a historical perspective, they tend to be above politics.)

The less-developed countries are now spending a smaller portion of the gross national product on social expenditures than the developed countries. It may not be generally appreciated that some of the developed capitalist countries are devoting a greater portion of the national resources to social expenditures than are some of the welfare or even socialistic states which are less-developed. If social expenditures can be interpreted to express the principle "to each according to need," then the rich capitalist countries are more advanced in that respect than the less-developed welfare or socialist states.

A number of less-developed countries are, however, making a much greater effort in social expenditures today than was made at comparable levels of economic development in the developed countries. This points to a significant departure from traditional patterns of development, brought about by the explicit recognition of the need for "human materials" in the course of economic development, and by the new horizons of social goals quite aside from considerations of economic gain. The "demonstration effect" is apparently working even faster in the social sphere than in the economic sphere for some latecomers.

Methods Used in Evaluating Adequacy of Old-Age and Survivors Insurance Benefits. EDNA C. WENTWORTH, *Social Security Administration.*

The Bureau of Old-Age and Survivors Insurance has evaluated the economic resources of beneficiaries in three ways. First, the income of beneficiaries was compared with the amount required for a minimum level of living. In the earlier surveys, beneficiary income was compared with the cost of the Works Progress Administration budget, adjusted to the requirements of a single aged person and an aged couple. In later surveys, the public assistance budget was adopted as the yardstick. Two-thirds of the beneficiaries were found to have nonassistance incomes that were below their respective state public assistance budgets.

The second method was to determine how many beneficiaries were partially dependent during the survey year. Approximately half of the beneficiaries studied were partially dependent; more received help from their children than from public assistance agencies.

The third method of appraisal involved revisits to beneficiaries already surveyed to determine how their economic situation had changed during a given period of retirement. Two such resurveys pointed up the fact that the economic situation of OASI beneficiaries worsened with time, and dependency on relatives increased.

These findings have helped to sharpen the picture of the economic situation of OASI beneficiaries and have pointed up the importance of the role played by benefits as a resource in old age. They have also been of value in considering basic policy questions in the old-age and survivors insurance program.

Non-Additivities in a Latin Square Design. M. B. WILK, and OSCAR KEMPTHORNE, *Iowa State College.*

A brief history of the Latin square design is given. A generalization of the design is given to the case in which the rows, columns and treatments represented in the experiment are samples from populations of rows, columns and treatments respectively. A possible frame of reference for the interpretation of the experimental results is described. This leads to what is termed a "population model," various population parameters, means and components of variation which are of interest to the experimenter. No assumptions are made about additivity of experimental units and treatments. Results on expectations of mean squares in the analysis of variance are given in terms of quantities (denoted by Σ's), which result in a very concise description, and in terms of the components of variation. Biasses in the estimation of components of variation by means of the analysis of variance are discussed and assessed. Comparisons of randomised block designs and Latin square designs are given for the general case of non-additive treatments for random, mixed or fixed populations of columns. A generalization of the design is discussed. The mathematical machinery which is used to derive the results is presented briefly. Finally linear estimates and errors of estimates are discussed.

On the Power of Multiple Range Tests. R. LOWELL WINE, *Virginia Polytechnic Institute.*

In place of the usual power function used to compare two tests when testing the difference in a pair of means, a set of $n(n-1)$ power functions is introduced for testing differences between pairs from a set of n means. Before power expressions are formulated for a class of multiple range tests, the region of integration is carefully described. The power functions are written as an algebraic sum of integrals with limits involving the variables of integration. Each term in the power function is broken up into several parts and transformed, making the lower and upper limits of integration constant and infinite, respectively, so that each transformed integral may be evaluated by series expansion. Since the amount of calculation necessary to find power for four or more means is so great, an approximation method is introduced to find an upper and lower bound to power in the range tests.

Employment in Services. SEYMOUR WOLFBEIN, *Bureau of Labor Statistics.*

As we complete the first half of the present decade, the number of persons employed in the nonfarm sector of the American economy has moved past the 50 million mark. Perhaps the most significant development is that, under the impetus of rapid growth in the trade and service areas of the economy, there are now in the U. S. more persons engaged in the production of services rather than in the production of goods.

The fact that we now have more workers engaged in the productiln of services rather than goods is an important hallmark in the evolution of our standard of living. The tremendous gains in output which this country has witnessed during this century have been attained mostly through major advances in productivity which have prevailed so dramatically in some of the key goods producing sectors of the economy, particularly in agriculture and manufacturing.

Manpower and Expenditures in Industrial Research and Development. HELEN WOOD, *Department of Labor.*

The tremendous magnitude of private industry's research and development activities is indicated by their cost —estimated at $3.7 billion in 1953, or about two-thirds of the national expenditure for research and development in the natural sciences and engineering.

The Federal Government sponsored nearly two-fifths of all industrial research and development in 1953. The Department of Defense and the Atomic Energy Commission were the chief contracting agencies.

Two industries, electrical equipment and aircraft, far exceed all others in the scale of their research. The electrical equipment industry did $778 million worth of research work in 1953—including $444 million worth performed for the Federal Government. The aircraft industry's research activities cost $758 million, of which $640 million was federally financed.

About 157,000 scientists and engineers, or nearly 30 per cent of all those in industrial employment, were engaged in RD work in January 1954. The number of technical, administrative, and other supporting personnel employed in research and development was roughtly one and three-quarters times as large as the number of scientists and engineers. Thus, the total number of persons engaged in research activities in January 1954 exceeded 400,000.

Who Makes Designs? W. J. YOUDEN, *National Bureau of Standards.*

Randomised blocks and Latin Squares are examples of classical experimental designs that reflect the needs of agricultural experimentation especially in the matter of providing generous replication of the treatments. The extension of the area of application into the physical laboratories where greater precision is attainable and to expensive pilot plant experimentation prompted the development of designs calling for a minimum of replication. Chain blocks, partially balanced incomplete blocks, and the Box designs are examples in points. Circumstances exert a determining influence in the evolution of experimental designs.

Decision Theory Aspects in the Design and Analysis of Engineering Experiments. J. ARTHUR ZOLLNER and PAUL GUNTHER.

Although Wald's recent advances in decision theory have gone far toward making statistical techniques conform to the real world, there still exists a considerable gap between theory and practice. In this paper, a general framework of decision, applicable to engineering investigations, is presented. The general evolution of the engineering investigation is conveniently divided into (1) a groping stage, (2) a firming up state, and (3) a precising stage. In each of these the statistical problems are different. For (3) Box's methods of response surface fitting may be quite feasible. In (1), the investigation is usually not sufficiently well-defined to use many statistical methods. In (2), the balanced statistical design has most usefulness, but should be modified according to the extent of a priori information.

Part I takes up an actual machinability problem where the immediate goal is to provide effective methods of grinding new alloys in such a way that induced stresses can be controlled. The example is flavored by the a priori knowledge of the experimenter and the utilities of certain decisions he must make. The important decision elements are thereby suggested in a pragmatic manner. In Part II these elements are formalized: There are two aspects to any investigation. (1) The *physical* setup which gives rise to facts and data and (2) the *human* role which gives rise to utilities. These are merged into a *model* which serves to organize the facts into a useful system. At any moment in time, the experimenter must decide either (1) get more facts, or (2) improve the model with existing facts, or (3) make the terminal decision. A number of suggestions are made regarding improvements in modern statistical practices.

STATISTICAL ABSTRACTS

All communications concerning this section should be addressed to the Abstracts Editor, Professor W. L. Smith, Department of Statistics, University of North Carolina, Chapel Hill, North Carolina.

Armitage, P., "Tests for linear trends in proportions and frequencies," *Biometrics*, 11 (1955), 375–86.

Thorough consideration is given to the $2 \times k$ contingency table with the k categories having a natural order (such as succeeding, but possibly unequal time periods, or severity of a lesion on a +, ++, +++, scale, etc.). In this case the common χ^2 statistic gives an inappropriate test since its value is unchanged by permutation of the frequencies among the k categories, but the sense of the table is not. Methods investigated include the use of scores and Kendall's τ (equivalent to the Wilcoxon test with tied ranks). Examples are given. L. E. Moses, *Stanford University*.

Barton, D. E., "A form of Neyman's ψ_k^2 test of goodness of fit applicable to grouped and discrete data," *Skandinavisk Aktuarietidskrift*, 38 (1955), 1–16.

This paper gives a natural adaptation of Neyman's ψ_k^2 test to the case of grouped data, which could be much easier to use under some circumstances. It also applies to discrete populations, although then it doesn't look so natural.

In the case that k is one less than the number of groups, Barton's test is the usual χ^2 test. In the case of equiprobable groups, it is the same as the test given by E. L. Scott (*The Astrophysical Journal*, 109, 1949).

Some discussion of power is given, particularly for $k=1$, 2. The loss of power due to grouping appears small for $k=1$, 2, 3 and at least 10 groups. J. W. Pratt, *University of Chicago*.

Bennett, B. M., "On the cumulants of the logarithmic generalized variance and variance ratio," *Skandinavisk Aktuarietidskrift*, 38 (1955), 17–21.

The "generalized variance" is the determinant of the sample variances and covariances of a multivariate sample. For sampling from a p-variate multivariate normal population, the author derives general expressions for the cumulants of the logarithm of the "generalized variance." The mth cumulant is the sum of p terms, each an mth derivative of the logarithm of the gamma function. D. L. Wallace, *University of Chicago*.

Bergstrom, A. R., "An econometric study of supply and demand for New Zealand's exports," *Econometrica*, 23 (1955), 258–71.

A system of 27 equations is postulated as a model for estimating the parameters of the New Zealand (N.Z.) supply and United Kingdom (U.K.) demand relations for New Zealand's main export products: dairy products, lamb, and mutton. In addition to the N.Z. supply and U.K. demand relations, the system includes N.Z. demand relations, equations explaining N.Z. livestock numbers, N.Z. labor supply and demand relations, a N.Z. income generating relation, and various margin and adjustment equations. Estimates of structural coefficients are presented for the three U.K. demand relations, the three N.Z. supply relations, and two N.Z. livestock numbers relations. Direct least squares (L.S.) and two sets of limited information estimates (L.I.) differing with respect to the number of over-identifying restrictions ignored are presented for each relation estimated. A priori considerations, conventional statistical measures of precision, and comparison of "predictive performance" with that of two "naive" models combine to form the basis for appraisal of L.I. and L.S. results. Ivan M. Lee, *University of California, Berkeley*.

Bliss, C. I., "An introductory course in biometry for graduate students in biology," *Biometrics*, 11 (1955), 213–24.

This paper discusses the author's course in biometry as given for the past eleven years at Yale University, and presents the results of a survey of his former students. Some points of interest concern the extensive, rather then the intensive approach; training the student to recognize situations in which expert statistical advice is necessary; the use of individual tutorials; the great importance of laboratory work; and the "delayed response" in the study of statistics. Richard L. Carter, *University of North Carolina*.

Box, G. E. P. and Youle, P. V., "The exploration and exploitation of response surfaces: an example of the link between the fitted surface and the basic mechanism of the system," *Biometrics*, 11, (1955), 287–323.

This is a sequel to an earlier article of the same general title published in *Biometrics*. The previous article described a number of applications of newly developed techniques for the study of response surfaces. The present article shows how study of the form of the empirical surface can throw important light on the basic mechanism operating and can thus make possible developments in the *fundamental theory* of a process. This idea is illustrated in some detail with an example previously discussed only from the empirical standpoint. A theoretical surface, based on reaction kinetics is now derived, rate constants are estimated from the data, and the theoretical surface is compared with the empirical surface previously obtained. It is then shown how the canonical variables of the empirical surface can relate to the basic physical laws controlling the system. In this connection the problem of suitable choice of metrics for the variables is discussed. In a final section some general remarks on the process of scientific investigation

are appended. [Author's summary] L. E. Moses, *Stanford University.*

Cornish, E. A., "The multivariate t-distribution associated with a set of normal sample deviates," *Australian Journal of Physics*, 7 (1954), 531–42.

——, "The sampling distributions of statistics derived from the multivariate t-distribution," *Australian Journal of Physics*, 8 (1955), 193–9.

If $x_1, \cdots, x_p$ have a multivariate normal distribution with zero means and covariance matrix $\sigma^2 R$ known except for the factor σ^2 and if s^2 is an independent mean square estimate of σ^2 based on ν degrees of freedom, then $(t_1 = x_1/s, \cdots, t_p = x_p/s)$ have a multivariate t-distribution. In the first paper, marginal and conditional distributions and distributions of linear and quadratic functions of multivariate t are derived in the special case in which the x's are deviations from the sample mean in an independent normal sample. Most of the derivations are valid for any matrix R.

In the second paper, the author derives the sampling distributions of studentized sample means, variances and covariances in a sample of n observations from the p-variate multivariate normal distribution given above. The observations are all studentized by a single independent s^2. Equation (4) of p. 194 for the joint density function of the studentized observations is incorrect; it would be correct for independent s^2's for each sample. But the sampling distributions are correct, being derived from the multivariate normal distribution without using the multivariate t-distribution. Albert Madansky, *University of Chicago.*

Cox, D. R., "Some statistical methods connected with series of events," *Journal of the Royal Statistical Society* (*B*), 17 (1955), 129–57.

The paper gives methods for the statistical analysis of events occurring haphazardly in space or time. For events occurring at random at a rate λ, where λ is a product of unknown parameters, the use of a logarithmic transformation to achieve additivity is described. Various types of non-randomness are considered and tests are given, several of which are based on $I(x)$, the ratio of the variance of the number of events in a time x to the corresponding variance for a random series with the same mean interval. A method is given for the estimation of a component of variance when the rates of occurrence of the events belong to some population. The author also gives a modification of Tippett's "Snap-round" method (or "Ratio delay") which allows estimates not only of the proportion of time for which a machine is running but also of the average rate of occurrence of stops and of the mean repair time per stop; instead of taking snap observations the operator watches for many short periods of given length and records the state of the machine as, for example, continuously running, stopped at the beginning but running at the end. E. S. Page, *University of Durham*, England.

Cox, D. R., "The statistical analysis of congestion," *Journal of the Royal Statistical Society* (A), 118 (1955), 324–35.

A review of several results of queueing theory is given. Solutions are discussed in the cases of simple queueing and machine-minding with respect to certain special applications. The "rush hour" problem is discussed in an appendix. Irwin Miller, *Virginia Polytechnic Institute.*

Downton, F., "Waiting time in bulk service queues," *Journal of the Royal Statistical Society* (B), 17 (1955), 256–61.

"The queueing process considered is that in which customers arrive at random and form a single queue in order of arrival. They are served in batches, the size of the batch being either a fixed number or the size of the queue, whichever is the smaller. Assuming that the queue is in equilibrium, the Laplace transform of the waiting time distribution is derived." The mean and variance of the waiting time are given for service times distributed independently as chi-squared with an even number of degrees of freedom. E. S. Page, *University of Durham, England.*

Duncan, David B., "Multiple Range and Multiple F Tests," *Biometrics*, 11 (1955), 1–42.

The analysis of variance, in even such a simple case as a balanced one-way layout, poses a complicated multi-decision problem as soon as individual comparisons among means are to be investigated. A "p-mean protection level" is written $\gamma(1, 2, \cdots p) = 1 - \alpha(1, 2, \cdots p)$ and is defined as:

$$\gamma(1, 2, \cdots p) = Pr\{\text{Deciding } \mu_1 = \mu_2 = \cdots \mu_p \mid \mu_1 = \mu_2 = \cdots = \mu_p\}.$$

The well known difficulty with applying the "least significance different method" is that in setting $\gamma(1, 2)$ at .95 one simultaneously sets $\gamma(1, 2, \cdots p)$ at a much lower level. On the other hand if there are n means and one sets $\gamma(1, 2, \cdots n) = .95$, as in, say, Scheffe's procedure, the resulting value of $\gamma(1, 2)$ may be uncomfortably large—yielding low power against any particular alternative $\mu_i \neq \mu_j$. In this paper it is proposed to adopt "protection levels based on degrees of freedom." If γ_2 is chosen, then this scheme provides for choosing $\gamma_p = \gamma_2^{p-1}$. A test procedure based on this system of protection levels is presented. Tables of significance values (1% and 5%) are given. Optimum properties are claimed. Alternative methods are discussed. L. E. Moses, *Stanford University.*

Elfving, G., "Geometric Allocation Theory," *Skandinavisk Aktuarietidskrift*, 37 (1954), 170–90.

An experimenter is free to take one or more independent observations on any of a given set of vector random variables whose distributions depend on a finite number of parameters. The expectations of these observations are known linear combinations of the parameters, and the covariance matrix of each available random variable is known. Associated with each of the random variables is a known cost per observation. Subject to the restriction that the total cost of the experiment is fixed, the experimenter must decide what observations to take in order to minimize the variance of the least squares estimate of a given linear function of the parameters.

Representing the given linear function as a vector in a finite dimensional space the author

shows that the optimum choice of random variables can be characterized as the intersection of this vector with the boundary of a certain convex subset of the space.

An example in sampling theory with regression estimation is given. M. H. DE GROOT, *University of Chicago.*

Estes, W. K., "The problem of inference from curves based on group data," *Psychological Bulletin*, 53 (1956), 134–40.

" . . . the valid interpretation of group curves depends on the principles common to all problems of statistical inference. Although the form of a group mean curve does not determine the forms of the individual curves, it does provide a means of testing exact hypotheses about them. In each particular case, the procedure must be to state explicitly the hypothesis under test, and then to derive the properties that should hold for the averaged curve if the hypothesis is correct. If the predictions thus derived are in accord with data, the hypothesis remains tenable; if they are not, then the hypothesis can be rejected at some specified level of confidence. Utilized within this framework, the averaged curve can be expected to remain one of the most valuable techniques for the analysis of behavioral data, and in fact to increase progressively in value as mathematical and statistical research continue to enlarge our repertory of special devices for the handling of particular problems." JULIAN C. STANLEY, *University of Chicago.*

Gartaganis, Arthur J. and Goldberger, Arthur S., "A note on the statistical discrepancy in the national accounts," *Econometrica*, 23 (1955), 166–73.

Results of certain tests of statistical properties of the "Statistical Discrepancy" in the U. S. Department of Commerce national income accounts are reported. The questions explored are: (1) Can the observed values of the Statistical Discrepancy be regarded as temporally independent drawings from a normally distributed population? (2) Does the size of the Statistical Discrepancy vary systematically with the size of the Gross National Product? (3) Do the periodic revisions of the account affect the properties of the Statistical Discrepancy? Annual and quarterly estimates from the 1954, 1951, and 1947 National Income Supplements are subjected to test. Absolute dollar values as well as ratios of Discrepancy to GNP are analyzed. The Kolmogorov-Smirnov d statistic is employed in tests for normality while the von Neumann δ^2/s^2, Wallis-Moore χ_p^2, and correlogram analysis have been employed in exploring the question of temporal interdependence. Results of the various tests are summarized and briefly discussed. IVAN M. LEE, *University of California, Berkeley.*

Hamaker, H. C., "Experimental design in industry," *Biometrics*, 11 (1955), 257–86.

This paper presents a point of view about the use of, and presentation of results from, the analysis of variance. It is argued that the conventional analysis of variance table is useful to the statistician but not to his industrial client, who will profit more from a display of the observations, the fitted values, and the residuals, together with an estimate of error, both per observation and for the "constants" which have been fit. The advantages of this approach for the statistician (viz., looking at the residuals as a clue to adequacy of the model) are pointed out. A (presumably likelihood ratio) test for non-additivity due to Ward and Dick is illustrated; the procedure may be viewed as an extension of Tukey's test in the two way layout. L. E. MOSES, *Stanford University.*

Healy, M. J. R., "A significance test for the difference in efficiency between two predictors," *Journal of the Royal Statistical Society* (B), 17 (1955), 266–8.

A significance test (due to Hotelling) of which of two quantities x_1, x_2 is the more effective linear predictor of a dependent variate y is derived by a regression approach. "More effective" is used here meaning "with smaller residual variance about the regression line." The method is generalized to test for differences in residual variances for three or more predictors. E. S. PAGE, *University of Durham, England.*

Houston, George, "Farm wages in central Scotland from 1814–1870," *Journal of the Royal Statistical Society* (A), 118 (1955), 224–8.

With data recovered from reporters' accounts of a hiring fair held every year in a central Scottish county the author proposes two questions: Can trends in farm wages taken from this one county be representative of Scotland as a whole? How do these figures compare with those for other parts of Britain?

The author further supports the data from the hiring fair by a comparison with the records kept by one farm in the area. The comparison shows approximately the same long term trend.

In regard to the first question, Houston does not feel that the figures represent the true picture for the whole of Scotland. On the other hand, the second question can be partially answered. The figures from Scotland represent cash wages and since the farm workers were also paid wages in kind it is not practical to compare these figures directly with those for the rest of Britain which represent total wages. However, with the theory that changes in cash wages represent marginal fluctuations in total wages, the long term trends of the two sets of figures may be favorably compared. RICHARD A. STEWART, *Virginia Polytechnic Institute.*

Jessen, Raymond J., "Determining the fruit count on a tree by randomized branch sampling," *Biometrics*, 11 (1955), 99–109.

The author considers the problem, which occurs in crop estimation and some areas of biological and pomological research, of estimating (without picking) the total number of fruit on a tree. In the sampling model, the tree is considered as a hierarchy of successively branching systems, with the ultimate branches bearing the fruit. (A later modification allows for the occurrence of fruit in intermediate positions.) The sampling scheme consists of a randomizing procedure for deciding, at each fork, which one of the branches shall be followed; successive appli-

cations of this procedure, beginning at the first fork on the trunk, result in the selection of a sample set of fruiting branches, on which the fruit are counted.

The essence of the scheme is the assignment of probability weights to the branches at each fork prior to making a randomized choice among them. Methods are considered in which the probabilities are: (I) equal for each branch, (II) proportional to the number of sub-branches into which each branch divides, and (III) proportional to the cross-sectional area of each branch. Results of applying these methods to the same pineapple orange tree are given, and show that method (III) is to be preferred. RICHARD L. CARTER, *University of North Carolina.*

Jowett, G. H., "The comparison of means of sets of observations from sections of independent stochastic series," *Journal of the Royal Statistical Society* (B), 17 (1955), 208–27.

The author gives a large sample test for the mean of a section of stationary time series using an estimate of the standard error of the mean based on the serial variation statistics (i.e. depending on the squared differences between the sample values). The method is claimed to possess advantages over one using the sample autocorrelations. The ordinary analysis of variance test for the equality of the means of k equal samples is modified to apply to the means of sections of independent time series. The tests are generalized to apply to spatial, unevenly spaced, continuous, series and to some stable but nonstationary series. E. S. PAGE, *University of Durham, England.*

Jureen, L., "Long-term trends in food consumption: a multi-country study," *Econometrica*, 24 (1956), 1–21.

This paper focuses attention on the relevance of economic structural disparity between countries in analyzing the pattern of demand for food. Special attention is given to the influence of varying income level on demand elasticity. The consumption-income relations around which the analysis centers are simple hyperbolic demand curves fitted to data on per-capita calorie intake and real income for 16 western European countries and the United States. Two broad food aggregates, "animal products" and "all other food items," are segregated in this stage of the analysis. Comparison of prewar and postwar fitted multi-country relations and comparison of multicountry relations with results of studies for individual countries based on time series and family budget data provide the basis for appraising the relevance of income to intercountry comparisons of food consumption. In assessing the relevance of food prices, results of studies for individual countries as well as direct analysis of intercountry consumption and price data are employed. Procedures used are described and methodological considerations underlying procedural decisions are indicated.

Substantive results presented are regarded as suggestive of the relevance of income and food price relationships in intercountry comparisons and as an aid to qualitative judgments regarding consumption patterns and trends rather than as a basis for quantitative forecasts. IVAN M. LEE, *University of California, Berkeley.*

Kendall, M. G., "Further contributions of the theory of paired comparisons," *Biometrics*, 11 (1955), 43–62.

The situation, where eight people must elect three of their number is discussed in some detail. Each votes by naming three people. The situation varies somewhat on whether the voter's ballot does or does not order his three choices. From the results an ordering can be constructed, as well as a coefficient measuring the departure from perfect ranking. The matter is explored further; in fact, the Sonneborn-Berger system of breaking ties in chess tournaments is investigated, though not named; it is shown to be equivalent to squaring the matrix representing the wins, losses, and draws. Geometrical interpretations are offered. The case of several judges making all paired comparisons is studied. Finally, consideration is given to the case of "incomplete blocks" where each judge decides on only a subset of the possible comparisons. L. E. MOSES, *Stanford University.*

Kramer, Clyde Young, "On the analysis of variance of a two-way classification with unequal sub-class numbers," *Biometrics*, 11 (1955), 441–52.

Let n_{ij} denote the number of observations in the ith row and jth column of a two way classification. Similarly let $n_{i.}$ and $n_{.j}$ denote row and column totals of the n_{ij}. Further, let y_{ij} denote the cell mean of cell (i, j). It is proposed to construct sums of squares for rows (and columns) assuming no interaction, by using "marginal weighted means" defined as:

$$y_{i.}'' = \frac{1}{n_{..}} \sum_{j=1}^{q} n_{ij} y_{ij}$$

$$y_{.j}'' = \frac{1}{n_{..}} \sum_{i=1}^{p} n_{ij} y_{ij}$$

The method is not exact (as is the method of fitting constants) but it involves much less work. The author presents indications that if certain inequalities among the n_{ij} obtain, then the method will be more powerful than the method of weighted squares of means, another commonly used nonoptimum procedure. L. E. MOSES, *Stanford University.*

Kuh, Edwin and Meyer, John R., "Correlation and regression estimates when the data are ratios," *Econometrica*, 23 (1955), 400–16.

Starting from the approximate expression developed in an early paper by K. Pearson for the correlation between ratios with a common deflator (X/Z, Y/Z,) a condition on the coefficient of variation of the deflator (V_z) is developed for equality of the ratio correlation and the partial correlation of X and Y for given Z. The condition takes the form $V_z = r_{xz}V_x = r_{yz}V_y$ and it is shown that for this condition to hold it is necessary and sufficient that each of the main variables be linear homogeneous functions of the deflating variable. This condition also defines

the minimum of the ratio correlation with respect to V_s. The generalization of the condition on V_s to the case of more than two main variables is indicated.

The bearing of the development indicated above on the practice of deflation in econometric applications is discussed with reference to both cross-section and time series data where it is concluded that deflation in cross-section data is probably less hazardous as a rule. Furthermore, it is argued that the possible gain in efficiency resulting from removal of heteroscedasticity through deflation frequently provides a valid basis for cross-section analysis based on deflated values. Finally, the bearing of the results of this paper on appraisal of certain previous econometric studies (primarily Chenery's work with investment functions) is briefly discussed. (Note: Presumably, the variance specification intended on line 2, page 407, is $s_i^2 = s^2/w_i$). IVAN M. LEE, *University of California, Berkeley.*

Lane, K. F. and Andrew, J. E., "A method of labour turnover analysis," *Journal of the Royal Statistical Society* (A), 118 (1955), 296–314.

The authors give an alternative, based on the estimation of a "leaving function," to the usual labor turnover rate or index analysis of employee stability, and illustrate it with examples. Two methods of estimating the leaving function are also shown and the form of the stability curve is discussed. PAUL MINTON, *Virginia Polytechnic Institute.*

Lesser, C. E. V., "Production functions and British coal mining," *Econometrica*, 23 (1955), 442–6.

Coal production functions are fitted to combined regional and time series data (two sets of results, 11 years for, alternatively, eight and nine regions) employing transformed data in an attempt to remove trend common to all regions. The production relation is of the form $P = g_t L^{\alpha} C^{\beta}$, where P, L, and C represent measures of output, labor input, and capital input, respectively. The output and input measures in the production relation are ratios of each regional measure to the geometric mean over all years for that region. For the estimation of α and β, the logarithms of the variables for each region in each year t are expressed in deviations from the arithmetic mean of logarithms over regions for given t thus transforming the logarithmic variables to measures with zero mean. The coefficients α and β are then estimated by least squares. From the estimates of α and β, the common trend component, γ_t, in the logarithmic relation, $\log P = \gamma_t + \alpha \log L + \beta \log C$, is derived. The antilog of γ_t provides the measure of g_t. Results are presented and briefly examined. IVAN M. LEE, *University of California, Berkeley.*

Lomnicki, Z. A., and Zaremba, S. K., "Some applications of zero-one processes," *Journal of the Royal Statistical Society* (B), 17 (1955), 243–55.

A discrete parameter stochastic process is called a zero-one process if the random variables composing it take only the values 0 and 1. With any discrete parameter process $\{x_\tau\}$ are associated zero-one processes: $Ux_\tau = 1$ if $x > 0$, 0 if $x_\tau \leq 0$ (the "basic" zero-one process for $\{x_\tau\}$); $U\Delta x_\tau$ (the uprun process); $U\Delta U\Delta^k x_\tau$ (the upcross process of order k) where Δ is the difference operator. When $\{x_\tau\}$ is Gaussian, estimates of some parameters, including the autocorrelation function, are obtained from the associated zero-one processes. E. S. PAGE, *University of Durham, England.*

McIntyre, G. A., "Design and analysis of two phase experiments," *Biometrics*, 11 (1955), 324–34.

In experiments where the effects of treatments have to be determined by a subsequent stage of assaying it is important to consider design in the assay phase in relation to the design in the primary phase. Examples of designs which enable the assay totals deriving from material in first phase plots to be analyzed for treatment and error mean square according to the first phase design are discussed. [Author's summary] L. E. MOSES, *Stanford University.*

E. S. Pearson, J. Wishart, G. A. Barnard, M. S. Bartlett, D. G. Champernowne, and D. J. Finney, "A discussion on the teaching of mathematical statistics at university level," *Journal of the Royal Statistical Society* (A), 118 (1955), 193–223.

This paper contains the views of six leading English statisticians on the teaching of mathematical statistics at the University level as well as some further discussion. M. S. Bartlett's contribution contains an analysis of the program at the University of Manchester including a discussion of the courses offered at that institution. Four appendixes give outlines of the programs in statistics and courses offered at University College, London, University of Cambridge, Imperial College of Science and Technology, London, University of Oxford, and London School of Economics. JOHN E. FREUND, *Virginia Polytechnic Institute.*

Philips, Almarin, "The variation of technical coefficients in the antifriction bearing industry," *Econometrica*, 23 (1955), 432–41.

Individual plant data (1942–1946) are analyzed in an attempt to isolate factors which might help to explain temporal variation in the antifriction bearing industry input-output ratio. Various measures constructed from ratios of selected inputs to output for individual plants are regarded as suggesting temporal behavior in individual plant ratios similar to that in the industry aggregate. Stable relationships among individual plant input-output ratios suggest that such factors as changes in product mix and differential effects of changing technology cannot be regarded, in the period studied, as having generated the temporal pattern observed in the industry ratio.

Although considerable homogeneity in temporal behavior of plant input-output ratios is suggested, substantial variability in ratios at a given point in time is observed. Individual plant data from the 1947 census are analyzed in an effort to identify some of the factors generating interplant variability. The procedure here is

illustrated with reference to the ratio of a particular input (metal consumed) to output (shipments). Essentially, the procedure involves an attempt to detect relationships between measures of such factors as heterogeneity of product, degree of vertical integration, size of plant, and plant technology and residuals from regression, linear in logarithms, of metal consumption on shipments. IVAN M. LEE, *University of California, Berkeley.*

Porter, R. A., "Operations analysis: experience and concept," *Skandinavisk Aktuarietidskrift*, 38 (1955), 37–52.

Introductory, expository article.

Redfern, Philip, "Net investment in fixed assets in the United Kingdom 1938–53," *Journal of the Royal Statistical Society* (A), 118 (1955), 141–92

The purpose of the paper is to estimate net investment in fixed assets in the United Kingdom, that is, that part of investments that increases the amount of fixed assets as opposed to that part which is used to replace obsolete and worn out previously existing assets. In order to accomplish this, a linear depreciation rate is assigned to existing fixed assets and this depreciation is subtracted from gross investment in fixed assets.

The first part of the paper explains the procedure, giving an artificial example and discussing the various factors that enter into the theory. The second part consists of actual estimation of net investment in fixed assets for various industries. R. J. FREUND, *Virginia Polytechnic Institute.*

Reierøl, Olav, "Tests of linear hypotheses concerning binomial experiments," *Skandinavisk Aktuarietidskrift*, 37 (1954), 38–59.

The paper discusses testing the hypothesis that each one of an assigned set of linear functions of expected frequencies is zero, given binomial data classified according to one or more criteria. Explicit chi-square tests of certain hypotheses relevant to data in one way, $r \times s$, $r \times s \times t$, 2^k, or $3 \times 2 \times 2$ classification are presented. It is pointed out that these tests are asymptotically equivalent to (asymptotically optimum but computationally difficult) likelihood ratio tests. The derivation of the tests and of their asymptotic property is based on Neyman's general theory of the chi-square test. A summary of Neyman's theory is included. R. R. BAHADUR, *University of Chicago.*

Roos, Charles F., "Survey of economic forecasting techniques," *Econometrica*, 23 (1955), 363–95.

A survey article given over primarily to business forecasting techniques. IVAN M. LEE, *University of California, Berkeley.*

Rutherford, R. S. G., "Income distributions: a new model," *Econometrica*, 23 (1955), 277–94.

A model of the distribution of the logarithm of income is sought having "explanatory" significance. From initial simplifying specifications regarding rates of entry and mortality and properties of the "shock system" in successive years, the form of the distribution resulting is Gram-Charlier Type A. The initial simplifying specifications are later relaxed and the effects of relaxation examined. A tentative method of fitting the distribution to income data is outlined. Finally, fitted distributions for selected countries in selected years are presented and the values of the resulting parameters briefly considered from the point of view of economic plausibility. IVAN M. LEE, *University of California, Berkeley.*

Schultz, E. F., Jr., "Rules of thumb for determining expectations of mean squares in analysis of variance," *Biometrics*, 11 (1955), 123–36.

Three rules are given which enable one conveniently to write down the expectations of all mean squares in any balanced analysis of variance under either the components of variance model or the mixed model. Several illustrative examples are given. L. E. MOSES, *Stanford University.*

Stuart, Alan, "The asymptotic relative efficiencies of tests and the derivatives of their power functions," *Skandinavisk Aktuarietidskrift*, 37 (1954), 163–9.

Two measures of asymptotic relative efficiency of two tests that have been used are the limit of the ratio of sample sizes necessary to achieve the same power against an alternative hypothesis (that moves toward the null hypothesis as the sample size increases) and the limit of the ratio of the derivatives of the power functions at the null hypothesis. Under specified conditions, the first measure is shown to be a power of the second measure, so that the two methods of measuring efficiency are essentially the same. D. L. WALLACE, *University of Chicago.*

Theil, H., "Recent experiences with the Munich business test," *Econometrica*, 23 (1955), 184–92.

The Munich Business Test (MBT), a continuing survey of around 5,000 firms in Western Germany, provides monthly information regarding direction of expected and actual changes in selected economic variables (production, price, stocks, etc.). The present article reviews some of the analyses of MBT data from the point of view of "usefulness" for econometric research. Attention is given to: (1) analyses dealing with the relationship of MBT data to official index numbers of the same period, regarded as tests of "reliability" of MBT and (2) analyses of interrelations of survey results themselves. An example of the latter is the analysis of disconformity described briefly as follows: A simple auxiliary discrete distribution is defined for three values, -1, 0, 1, having probabilities equal to the fractions of firms reporting a decrease, no change, and an increase, respectively, in a given economic variable. The mean of this distribution is the "balance" and the variance termed (Theil and Cramer) the "disconformity coefficient." Medians of time series of disconformity coefficients for a number of variables (expected and actual) for six stages of the German leather and shoe industry are presented. Internal analysis of survey data is further illustrated by equations relating monthly balances of test variates, postulated for testing relationships between central

tendencies of microvariables. Least-squares results are given for three stages of the leather and shoe industry relating balances in expected buying prices to balances in lagged actual buying prices in the same stage and lagged actual selling prices in "higher" stages. IVAN M. LEE, *University of California, Berkeley.*

Thom A., "A statistical examination of the megalithic sites in Britain," *Journal of the Royal Statistical Society* (A), 118 (1955), 275–91.

A geometrical and statistical study is made of 250 megalithic sites in England and Scotland. The sites consist mainly of circles of standing stones, of rows of standing stones, here called alignments, or of a combination of the two. Many circles have one or more outliers, i.e. single upright stones outside the ring.

In this paper, the three following points are examined: (1) The geometry of the "circles"; (2) The possibility that a common unit of length was used; (3) The possibility that the alignments and the outliers had an astronomical significance. It was found that many of the circles follow certain geometrical designs; that a universal unit of length was used in setting these out on the ground and that many of the constructions carry indications of astronomical uses beyond those associated with the sun. Numerous tables and diagrams are submitted to support these conclusions. A rigid statistical approach is lacking. LEO LYNCH, *Virginia Polytechnic Institute.*

Tukey, John W., "Reply in 'Queries'," *Biometrics*, 11 (1955), 111–3.

Earlier Tukey gave a method for isolating from the error sum of squares in a two way table a component with one degree of freedom which may be used to test the assumption of additivity. The method is generalized here to an arbitrary design falling into the framework of model I, and is illustrated for a latin square. The procedure is as follows.

Let x_α denote an observation whose expected value under the model is μ_α. Let X_α denote the least squares estimate of μ_α. (Thus in a latin square $X_{ij(k)} = x_{i\cdot\cdot} + x_{\cdot j\cdot} + x_{\cdot\cdot k} - 2x_{\cdot\cdot\cdot}$). Let $z_\alpha = A(X_\alpha - B)^2$ where A and B may be any constants chosen for arithmetic convenience. If one then carries out an analysis of covariance on the observations x_α using the z_α as regression variables, the sum of squares for regression has "one degree of freedom for non-additivity" and may be tested against the error. L. E. MOSES, *Stanford University.*

Ura, S., "On the power function of Welch's test procedure in the two sample problems," *Reports of Statistical Application Research, Japanese Union of Scientists and Engineers*, 4 (1955), 1–13.

Consider the two sample test procedure in which the variance of $\bar{X} - \bar{Y}$ is estimated by $(s_1^2/n_1) + (s_2^2/n_2)$ and the distribution of the test statistic is approximated by a t-distribution. The type I error of this procedure has been shown to vary much less with the ratio of the population variance than does the type I error of the usual two-sample Student's t-test (e.g. Welch, B. L., *Biometrika*, 36 (1949), 293).

In this paper, the power function of this approximate test is compared with the power function of two-sample Student's t test when the population variances are equal. Most of the comparisons are based on expansions of the power functions in power series in the reciprocal degrees of freedom. The power functions are shown to differ by at most 0.01 for a large range of values of the two sample sizes. D. L. WALLACE, *University of Chicago.*

West, John H., "An analysis of 162,332 lottery numbers," *Journal of the Royal Statistical Society* (A), 118 (1955), 417–26.

The randomisation procedure used in Southern Rhodesian lotteries is described and then the numbers generated in 67 of these lotteries are analyzed statistically. The distribution of the digits of which these numbers are comprised is found to conform to a rectangular frequency distribution with respect to its average but not with respect to its shape. RICHARD G. CORNELL, *Virginia Polytechnic Institute.*

Whittle, P., "Some distribution and moment formulae for the Markov chain," *Journal of the Royal Statistical Society* (B), 17 (1955), 235–42.

The distribution of the total number of transitions between the states of a Markov chain is derived and is shown to be similar to one based on a set of independent multiomial distributions. The author then proves directly the asymptotic distribution of Bartlett's goodness of fit statistic, and calculates the asymptotic likelihood and moments of the number of particular transitions and of the transition probability estimates. E. S. PAGE, *University of Durham, England.*

Wilson, Ross, "The Scotch whiskey industry," *Journal of the Royal Statistical Society* (A), 118 (1955), 345–62.

Following a brief discussion of the methods of manufacturing Scotch whiskey, this paper is devoted to a detailed review of the records of the industry, principally since 1939. A long series of tables deals with such aspects as: numbers of distilleries, production of ethyl alcohol, production and stocks of whiskey, quantities of ingredients used, domestic consumption of whiskey, export volumes and revenues. A. E. GARRETT, *Virginia Polytechnic Institute.*

Youden, W. J., and Hunter, J. S., "Partially replicated latin squares," *Biometrics*, 11 (1955), 399–405.

If interactions exist among some sets of treatments which are used as rows, columns and treatments of a latin square, the error will be overestimated, the entire nature of the treatment effects will be misconstrued and the experimenter may be deceived. The authors propose a check on the assumption of no interactions by taking duplicate measurements for *one* of the "treatments" corresponding to letters. They give formulas for convenient analysis of the design and illustrate the procedure on some data which suggest that the method can be useful indeed. L. E. MOSES, *Stanford University.*

BOOK REVIEWS

The Economic Report of the President, January 1956. Washington, D.C., Government Printing Office, 1956. Pp. x, 238. Single copy, 70 cents. Paper.

See review article by Arnold C. Harberger, pp. 454–60 in this issue.

Elementary Statistics for Students of Social Science and Business. *R. Clay Sprowls.* New York: McGraw-Hill Book Company, Inc., 1955. Pp. vii, 392. $5.50.

HRAND SAXENIAN, *Harvard University*

THIS text departs from the traditional approach to elementary statistics by introducing at the very beginning the basic concepts of statistical inference and decision making under conditions of uncertainty. This is accomplished through the use of problems which have an intuitive appeal to the nontechnically trained student. As Sprowls develops the statistical material which is now handled in almost all elementary textbooks he carefully and clearly keeps the emphasis on principles of inference, the ideas of hypotheses, risks of error, and the evaluation of these risks in terms of the operating characteristics of a statistical test. Throughout the development of the book the student is given ample practice in the analytical formulation of practical problems bearing on statistical inference in the discrete case, in the continuous case, in simple linear regression and correlation, and in the analysis of time series and index numbers. Time series are treated as random variables and statistical tests are introduced for the detection of systematic components before the estimation of these components. In correlation, emphasis is upon the regression model and the estimate of confidence bands.

Sprowls addresses the text to "ordinary citizens who will be exposed to statistical statements, arguments, and reports. The student is trained to evaluate these reports and to ask the questions that will prevent his being misled. He is taught to ask questions about the hypotheses, the random sample, the alpha and the beta, and the operating characteristics." The reviewer feels that the author does accomplish his avowed purpose of simplifying the mechanics of the statistics and raising the standards of student instruction by developing in the nontechnical student the use of technical statistical terminology and attempting "to teach some abstract reasoning." It is the reviewer's opinion that, while the author writes clearly and informally, a great burden is nevertheless still placed on the instructor who uses this text to support the early introduction of technical terminology and the concepts of statistical inference which are more complicated and rigorous than the ordinary nontechnical student has had to cope with. Again, whether the author accomplishes his purpose of teaching some abstract reasoning will depend in large part on the instructor's handling of the material developed for him in the text. The text itself is an excellent vehicle for this purpose, but the reviewer does not feel that in itself it can accomplish this broader purpose. The reviewer feels further that while the emphasis on the inferential aspects of statistics and sampling is certainly a sound one, the more conventional method of first introducing descriptive statistics (which in this text is handled half way through the book) helps the nontechnical student to orient himself to the more complicated analytical concepts which are stressed here. The reviewer also feels that while the early discussion of the concept of probability is well handled, the relationship between the concept of probability and the binomial and normal distribution curves could have been emphasized in a more basic manner.

The reviewer found the manner of presentation to be clear and the qualitative discussions stimulating and rather more complete than in other elementary texts, yet

not unnecessarily wordy. The many problems within each section seem to be exceptionally well chosen for the purpose of broadening the students' view of the wide range of applicability of the methods introduced to him.

There is quite a bit of cross reference between various sections of the text which should help to give the beginning student an integrated picture of the basic problems of statistical inference as he moves from one topic to another. There is also consistent emphasis on, as well as a chapter devoted to, statistical research methodology which should have a direct personal appeal to the student so that analytical statistics should become a practical part of the interested student's thinking. It is the reviewer's opinion that this text may help to give some interested students the incentive to continue the study of statistics and should help to give all students who manage to get through it a fuller appreciation of both the advantages and limitations of mathematical reasoning in everyday problems.

Statistics. *L. H. C. Tippett.* New York: Oxford University Press, 1956. Pp. vi, 224. $1.20.

According to the preface of this excellent little book, the first edition of which was published in 1943 and reviewed in this *Journal*, Vol. 40 (1945), pp. 536–8, by Ernest Nagel and John H. Smith,

> This edition has been revised to take care of [recent] developments [in statistics] and to describe things as they are today; and much of the original illustrative material has been replaced by more up-to-date material. The general scope of the book remains unchanged.

W.A.W.

Analysis of Confounded Factorial Experiments in Single Replications—Technical Bulletin No. 113. *F. E. Binet, R. T. Leslie, S. Weiner, and R. L. Anderson.* North Carolina Agricultural Experiment Station, 1955. Pp. 64. Paper.

M. B. Wilk, *Princeton University*

This technical bulletin is concerned, mainly, with the design and analysis of factorial experiments where confounding is either necessary or desirable and where only single replications are employed. Against this background the authors present recommended plans and procedures for analyses for various situations ranging from the 3×2 to the 7×2×2 and the 7×5, including forty factorial structures in all. "For some of the combinations, new designs are set out, while for others, both new and old designs are discussed." Fractional factorials are not considered.

The document is divided into two main parts: Part I, 17 pages, on "Background and General Procedure"; Part II, 40 pages, on "Presentation of Plans and Analysis."

Part I deals very briefly with assumptions and justifications for the analyses and designs described and gives some detail on computational methods for determining the bias and non-orthogonalities due to confounding and for calculating the analysis of variance. An example is worked out. The mainspring of the procedures recommended is the use of "the high degree components of all interactions (irrespective of the order of the interactions) . . . as error variance estimates." In consequence there is the restriction that "if there are more than two treatments (levels) for a given factor, these treatments can be ordered on a continuous scale." A further explicit assumption is "that there is no block-treatment interaction."

The method of analysis presented is based on the computation of single degree of freedom contrasts which are "orthogonal polynomial" contrasts (as for example, the linear, quadratic, linear, linear×linear, and quadratic×linear contrasts in a 3×2

experiment.) The computational procedures are due to Yates and Bainbridge et al.*

Part II consists of tabulations of plans for different confounding systems for the range of factorial structures considered. Accompanying each plan is a table summarizing the pattern of confounding of block contrasts with degree-component contrasts.

The more serious deficiencies in this document are associated with Part I. No justification (except general reference to other works) is given for the procedures recommended. Nor is any discussion given of possible pitfalls or alternatives. Specifically, for justification of the use of high degree components of all interactions to estimate error the reader is referred to Box and Wilson; the question of the meaning of the usual linear, quadratic, etc. components when the levels of a factor are not equally spaced is not mentioned; the effect of non-additivities of blocks and treatments is not discussed; in the presentation of assumptions, no separation is made between statistical assumptions on behavior of fluctuations and those assumptions on the model which depend on the mechanics of the experimental situation.

Other shortcomings are inadequate definition of notations and insufficient systematization in presenting detailed computing instructions. The novice would very likely have many difficulties in following the directions given, in spite of the attempt to give operating detail. In part this is due to heavy reference to other works at key points.

Part II is also not self-sufficient in that many plans dealt with are not reproduced but reference is made to where these can be found, and only the confounding patterns are tabulated.

In summary, this bulletin contains some new plans for confounded factorial experiments and useful tabulation of confounding patterns for new and many old plans. The document is not in any way comprehensive or definitive either in enumeration of plans, justification for methodology, or specification of computing procedures. Despite such shortcomings considerable effort is represented and the bulletin would be a useful addition to the library of most experimental statisticians.

Principles of Auditing. *Arnold W. Johnson.* New York: Rinehart & Company, Inc., 1955. Pp. x, 391. $6.50.

ROBERT M. TRUEBLOOD, *Touche, Niven, Bailey & Smart*

JOHNSON'S recent text on auditing is distinguished in that it includes a chapter on the application of statistical sampling methods to auditing. This inclusion is very probably a "first" in auditing literature; the author and the publisher are both to be commended. John Neter, presently of the University of Minnesota and a prolific writer on the subject of statistical sampling for accounting purposes, is the author of the chapter on sampling.

There are many who decry the failure of professional accountants to endorse statistical sampling enthusiastically. But perhaps the full depth, profundity, and interdependence of an auditor's evaluations and judgments are not completely understood by some of the statisticians who insist that full-scale adoption of statistical sampling in auditing is long overdue. This is not said in order to belittle in any way the good hard thought which is forced upon the auditor by the challenge of formulating quantitative measurements. Nor is it contended that quantitative measurements would not, where they are practical, significantly improve auditing practice.

* Bainbridge, J. R., Grant, Alison M., and Radok, U., "Tabular analysis of factorial experiments and the use of punch cards," *Journal of the American Statistical Association*, 51, 149–58, 1956. (This paper was not yet published at the time the bulletin under review was written.)

The auditor's problem is simply a bit more complex than most laymen comprehend.

The auditor works in a world of complex business systems, in which there are many interrelated variables constantly at play. As Neter explains, the estimation of the relative proportion of vendor's invoices involving clerical error can be simply appraised by acceptance sampling methods. However, even in so simple a situation, it is not at all clear that acceptance sampling is the answer. Quite possibly, the auditor might wish to estimate the dollar magnitude of error as well as the error rate—in which case survey sampling would be a much more useful means to the end. Even further, the critical factor to the auditor might well be the relationship of the invoice error rate (a factor of relatively minor interest, in itself), to a large array of other circumstances, conditions, error rates, control devices, and so on. Although estimation of a single characteristic of the invoices can be established readily by statistical means, the literature on auditing applications of statistical sampling simply does not generally go into the business of relating the significance of a particular error estimate to the entire complex of the total business system which is necessarily involved.

More power and much luck to Neter, and to other accountants and statisticians who are truly convinced and genuinely sincere about the fruitfulness of bringing statistical sampling to bear upon auditing problems. The cause will make more haste, however, if statisticians do not inadvertently represent their techniques as being more flexible and more broadly useful—in the over-all sense—than they really are.

Table of the Descending Exponential, $x=2.5$ to $x=10$. *National Bureau of Standards, Applied Mathematics Series 46.* Washington, D.C.: U.S. Government Printing Office, 1955. Pp. v, 76. 50 cents. Paper.

"The descending or negative exponential e^{-x} is tabulated in this publication from $x=2.5$ to $x=10$ at intervals of .001 to 20 decimal places. The table provides greater accuracy than has heretofore been available in this range of the argument and hence facilitates interpolation. . . . The table in this volume is a supplement to the tables of the exponential function e^x that were prepared in 1939 by the New York Mathematical Table Project under the Bureau's scientific sponsorship, Applied Mathematics Series 14. In the earlier volume the descending exponential is given from $x=0$ to $x=2.5$ at intervals of .0001 to 18 decimal places and for $x=1$ to $x=100$ at intervals of 1 to 19 significant figures."

W. A. W.

Correlated Random Normal Deviates. *E. C. Fieller, T. Lewis, and E. S. Pearson.* Tracts for Computers, No. XXVI. New York: Cambridge University Press, 1955. Pp. xv, 60; $2.00.

According to the Introduction, "These tables give in effect 3000 random pairs from each of nine bivariate normal distributions with respective correlation coefficients 0.1, 0.2, 0.3, · · · , 0.9. They have been compiled from Wold's tables of random normal deviates. The data are set out in the form

$$i\text{th line: } x_{0i};\ x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i}, x_{9i}\ (i=1, 2, \cdots, 3000),$$

where the ten values are random members of normal populations with means 0 and variances 1, and with correlation coefficients $\rho_t=t/10$ between x_0 and x_t ($t=1, 2, \cdots, 9$). All values x_0, x_t are given to two places of decimals (2 D). The values of the variate x_0 have been taken unaltered from Wold's tables. At the foot of each of the 60 pages of the table the following summary values for the 50 lines on that page

are given in addition: sums of variates (2 D), sums of squares of variates (4 D), sums of products with x_0 (4 D), sample correlation coefficients (4 D)."

W. A. W.

Handbook of Vital Statistics Methods, Studies in Methods Series F, No. 7. United Nations, Statistical Office. New York: Columbia University Press, 1956. Pp. iv, 258. $2.50. Paper.

HAROLD F. DORN, *National Institutes of Health*

THE *Handbook* is primarily a description of existing practices and procedures, rather than an exposition of recommended methods. The data were obtained in response to an inquiry sent to every country and nonself-governing area in the world in January 1950. Official replies were received from 65 sovereign nations and some 35 dependencies. The principal omissions are the Soviet Union and China, which together account for more than one fourth of the world's population.

The purpose of the *Handbook* is to assist countries in the development and appraisal of their vital statistics services by describing current practices, setting forth recommended standards, and presenting uniform concepts, definitions, and procedures. Vital statistics include records and statistics of live birth, death, stillbirth (late fetal death), marriage, and divorce. The practices and procedures described are, for the most part, those existing at the time of the survey in 1950. The data were analyzed by first outlining the basic elements of a national vital statistics system and then preparing a comparative summary of the practices in the 65 sovereign nations for each subject. While this makes it difficult for the reader to obtain a comprehensive view of the vital statistics system of a specific nation, it brings order and intelligibility to what otherwise would be a confusing mass of detail.

The *Handbook* is divided into 14 chapters and 4 annexes. The first three chapters discuss the present status and uses of vital records and statistics, together with governmental provisions for the registration of vital events and for the compilation of vital statistics. Registration procedures are described in a series of four chapters. The next five chapters take up the reporting process, the uses, definition, and classification of items included on statistical reports of vital events, methods of compilation, the program of tabulation, and the definition and computation of rates and indexes. The final chapter presents some methods for evaluating the vital statistics system with particular reference to completeness of registration and statistical reporting, and the measurement of the quality of the data.

The annexes include a chronology of important events in the development of civil registration of vital statistics, the definition of stillbirth in effect in 84 geographic areas on January 1, 1950, a selected list of references and a reprinting of *Principles for a Vital Statistics System* adopted by the Economic and Social Council of the United Nations in 1953.

The anonymous authors of the Handbook merit praise for the skillful manner in which they have organized and presented the mass of detailed information available to them. For the first time, the accumulated experience of most of the nations of the world in the registration, collection, compilation and tabulation of vital events has been made available in written form. Existing practices and procedures are compared with the international recommendations contained in *Principles for a Vital Statistics System* so that any nation can readily determine in what respects its system fails to measure up to the recommendations.

The usual rates and ratios used in the presentation of vital statistics are clearly described and, as a whole, more precisely defined than in some textbooks. The only exceptions are the definitions of the annual live birth rate specific for live birth order,

the annual birth ratio specific for parity, and the annual birth ratio specific for duration of marriage. Although the meaning should be apparent, the definition of these indexes lacked the precision characteristic of the definition of other rates and ratios.

On page 185 it is stated that the use of the population at the beginning of a time period will usually overstate a rate and that a more correct figure will be given by the average population exposed to risk throughout the period. This statement is true for central rates only. Rates may be defined for which the initial population is the appropriate denominator, e.g., the infant mortality rate.

However, these are very minor faults in an otherwise carefully prepared publication. The Statistical Office of the United Nations has performed a signal service for all who are interested in international vital statistics by the publication of this *Handbook*.

Perinatal Mortality in New York City. A Study of 955 Deaths. Analyzed and Reported by *S. G. Kohl*, M.D., Dr. P.H., Published for the Commonwealth Fund by Harvard University Press, 1955. Pp. xxi, 112. $2.50.

T. G. DONNELLY, *Dominion Bureau of Statistics*

THIS study reports on a sample investigation carried out in New York City into perinatal deaths (classified as deaths to stillbirths, to infants under 5 days, and to infants from 5 to 30 days.) The primary purpose of the investigation was to determine the actual causes of such deaths, and as such constitutes a study into the accuracy of birth and death registration procedures. Secondary purposes included assessing the preventability and responsibility for such deaths, and collecting relevant data on pre-natal and medical care, pathological findings, and obstetrical histories.

From a sample of stillbirth and death registrations for 1950 numbering nearly a thousand, detailed investigations were made on a large number of circumstances surrounding the deaths and the corresponding births. This entailed home visits, examination of clinical and autopsy reports, consultations with hospital physicians and pathologists, and finally an assessment as to actual cause of death, responsibility, preventability and adequacy of precautionary measures.

The most important statistical result is that birth and death certificates are shown to be far from reliable as sources of detailed information. On birth certificates corresponding to these deaths, discrepancies of a nontrivial nature were looked for on 10 selected items. An average of 2.3 nontrivial discrepancies per certificate was found, and only 6 per cent of the certificates were free of errors. For death certificates, major discrepancies in causes of death between death certificate diagnosis and clinical diagnosis occurred in about one-half of the cases.

A score of conclusions of the following nature are summarized at the end of the book. One-third of deaths, mostly of mature infants, were conceivably preventable, with the least room for improvement to be found in the deliveries at teaching hospitals, and in the work of trained obstetricians. The errors in causes on death certificates do not offset each other: there is a net under-reporting on certificates of deaths actually due to birth trauma and to malformations. Quite dependable material on actual cause of death is to be found in the hospital records, but this is usually not available at the time of filling out death certificates.

Much of the data collected and presented here on previous abortions, method of presentation and delivery, anaesthetics, pre-natal care, and other factors are of little use because the population of successful births was not sampled simultaneously with the population of perinatal deaths. However, this is the only weakness in an otherwise well-conceived research project that could serve as a model for similar investigations elsewhere.

Among statisticians the book will prove of greatest value to those charged with producing accurately or using intelligently vital statistics on perinatal mortality.

One's first reaction to this book is surprise at the poor quality of the statistical data on death certificates. For persons wishing to make comparative studies on infant mortality for different places or times, the implications of the state of affairs revealed here are far-reaching. Paradoxically, part of the reason for the low quality of statistics is likely to lie in the high quality of medical and public health services. With no really over-riding causes of infant deaths operative, doctors are often required to make quite detailed investigation to find what was the actual cause of death. Under these circumstances doctors may well feel that it is not worth the effort to make accurate infant death registrations: that there is no "story" here that should be brought to public attention through vital statistics reports.

The implications of this study for vital statisticians are many, but the author does not spell them out. It is to be hoped that the results of this study will exert a strong influence in the direction of improving statistical training in medical schools, tightening control over the quality of such mortality data, and increasing the co-operation between producer, compiler, and user of vital statistics.

Planned Migration: The Social Determinants of the Dutch Canadian Movement. *William Petersen.* University of California Publications in Sociology and Social Institutions. Volume 2. Berkeley and Los Angeles: University of California Press, 1955. Pp. x, 273. $3.50.

FRANK E. JONES, *McMaster University*

BETWEEN 1946 and 1954, approximately 102,000 persons emigrated from the Netherlands to Canada. In Petersen's view this movement, although deliberately stimulated by both countries and carried on under almost optimum conditions, provided inadequate relief for the Netherlands' extremely high population pressure and was smaller than Canada could safely absorb as part of its postwar immigration program. These inadequate results are attributed to planned migration which, characteristic of modern migration, involves the direct participation of national governments in contrast to the free migration typical of the 19th century.

Analysis of certain aspects of social structure in the Netherlands reveals that Roman Catholicism and Calvinism by virtue of strong power positions have successfully maintained the dominance of values usually associated with traditional rural communities. These traditionalist values provide a strong motivational basis for high fertility and severely restrict the range of solutions to the severe population pressure in the Netherlands by excluding birth control, the obvious rational solution to the problem.

Also, these values directly influence the degree of success of industrialization and emigration which, with land reclamation, are the three acceptable solutions to population pressure. In the case of industrialization, the dominant religious forces have successfully combatted the usual displacement of traditionalist values by urbanism. Hence, even if industrialization could achieve a more abundant economy, the important effects of its usual concomitants, particularly a low fertility rate, on population pressure have been lost. As these values offer no opposition to practices resulting in reduced mortality which also accompany industrialization, the persistence of a high fertility rate is all the more disastrous. Even though emigration is not contrary to traditionalist values, they foster a conservative attitude toward geographic mobility. Although the major religious groups have encouraged emigration, none is eager to lose adherents and the highest proportion of sponsored immigrants to Canada belong to the Orthodox Calvinist group which occupies the weakest power position

among the three major religious groups. Despite persistent attempts to make it attractive, emigration has fallen short of its goal of 40,000–60,000 emigrants a year.

In the postwar period, economic conditions have favored rapid population growth in Canada and the government has sought to supplement an increased birth rate with a large scale immigration program. There is not, however, unanimous agreement on the value of immigration and, as Petersen shows, the immigration rate is influenced by three major assumptions which underlie immigration policy and by the influence exerted by certain groups and population aggregates.

Canada's immigration policy is based on the assumptions that the most important occupational need is for agricultural workers, that Nordics, interpreted as British and Northwest Europeans, are most readily assimilable to Canadian ways, and that the number of immigrants should not exceed the absorptive capacity of the country. As Canada has been undergoing rapid industralization for some time, as there is little evidence to support the view that the British or Northwest Europeans do assimilate faster or as fast as other ethnic or nationality groups, and as absorptive capacity is a concept which continues to defy adequate empirical evaluation, Petersen advances the view that these assumptions simply provide either convenient rallying slogans for those who have a stake in increased or decreased immigration or provide effective devices whose very ambiguity allows concealment of the real issues in the power struggle.

Management and labor, English and French Canadians, and any others taking sides on this issue utilize these assumptions to defend their viewpoints even though the arguments are often irrelevant or cannot be decided by facts.

While the preference given to agricultural workers has facilitated entry for the Dutch, this assumption is based on an assumed need for persons interested in extensive farming whereas the Dutch tend to intensive farming. Similarly, the preference for Nordics gives the Dutch an advantage but the predominance of Orthodox Calvinists means the admission of a group which has strong religious motives for self-segregation.

This book contributes an important and refreshing approach to demographic analysis as well as an excellent study of the Netherlands-Canada migration. The need to regard values as crucial variables in population study is so convincingly demonstrated that one may well wonder why values have been neglected in this field.

Perhaps it is their elusiveness, apparent even in this admirable study, which leads to their neglect in favor of equations restricted to seemingly more tangible demographic and economic variables. Traditionalist values, despite their importance to the analysis, are only briefly described as "magical modes of thinking and behaviour" or as an antonym to Weber's concept of rationality and although the reader may infer from the discussion that large families, a small number of women in the labor force, and similar facts are regarded as signs of traditionalist values, there is no clear guide to their identification in empirical behavior.

While Petersen presents a convincing argument for the influence of values on demographic processes, his analysis does not completely support his thesis that values in relation to planned migration are responsible for the low volume of movement between the Netherlands and Canada. His analysis strongly supports the conclusion that the persistence of traditional values is largely responsible for the continued population pressure in the Netherlands as well as for the absence of a tradition to emigrate but the assumptions underlying Canadian immigration policy and various group pressures, although they may have restricted the flow of immigrants, appear to have facilitated the admission of Dutch immigrants. The failure of the movement seems to be more closely related to value barriers in the Netherlands than in Canada.

Petersen has demonstrated that planning may fail if formulated on the basis of false assumptions or in opposition to firmly entrenched values but this supports the thesis that faulty planning rather than "the nature of planned migration" is responsible for failure.

Future Population Estimates by Sex and Age. Report I, The Population of Central America (including Mexico), 1950–1980. Report II, The Population of South America, 1950–1980. *United Nations, Bureau of Social Affairs, Population Branch.* New York: Columbia University Press, 1954; 1955. Pp. 84; vi, 139. $0.70; $1.25. Paper.

ALCEU VICENTE DE CARVALHO,[1] *Institut Brésilien de Géographie et Statistique*

DURING its 7th Session the United Nations Population Commission recommended to the Secretary General that population projections be made "paying special attention to the need for such projections for underdeveloped countries, as aids in the planning of their economic and social development programmes." Following this recommendation the United Nations Population Division has now published in two separate reports its population projections for the countries of Central America, including Mexico, and South America, for the period 1950 to 1980.

Both reports follow the same general arrangment. The first gives a general explanation in five sections: methodology, assessment of the factors determining the degree of accuracy of the estimates, growth of population during the period 1920–1950 and forecast of its development 1950–1980, geographic distribution of the population in 1950 and 1980, and finally probable changes in the demographic structure. A main concern in the calculations was to secure results comparable for the several countries in such a way as to make possible regional totals.

The procedure consists of three principal steps: (a) securing data for estimates of the male and female population of the several countries by five-year age groups for the starting date of the projection, (b) calculation of the survivors of each of these five-year age groups at the end of each period of five years, according to certain hypotheses as to the future rates of mortality, and (c) calculation of the survivors, ages 0–4 years, of the children born in the course of each 5-year period using hypotheses on future birth rates.

The effect of international migration on the growth of population in the countries of Central and South America is not taken into account in the first approximation because of the obvious difficulties of forecasting reasonable and comparable figures for these countries. Nevertheless, considering that among the countries of South America are some for which immigration has been an important factor of growth, some material is presented in an appendix. As an illustration calculations are made for Argentina and Venezuela which provide useful illustrations of the consequences of immigration.

In the main calculations no difficulty arises in obtaining figures for the starting point; these are provided by the 1950 Census. For mortality the decline observed over a long period of time is recognized for each sex and age group. Mortara points out that this method implies considerable confidence in demographic uniformities but that such confidence is the condition of any forecast.

For births, past experience does not provide such a uniform trend. The publication classifies each country according to type, following the known trend of its past mortality rates. Then, according to the type of country, use is made of one of three hypotheses of future variation (high, medium and low) and a rate is fixed about which it is assumed that births will be specialized after a certain period. The last step is to establish the movement between present birth rates and the ultimate rate assumed.

[1] Translated by Nathan Keyfitz, *Dominion Bureau of Statistics.*

Mortara considers that though it is risky to assume that any forecast of births would be fulfilled, yet the writers have proceeded with great caution and with full awareness of the difficulty of the problem.

Leaving to one side the effect of international migration, the total population of Central America including Mexico, which was 34.6 million in 1950, will in 1980 be 64.5 million on the low projection, 74.0 million on the middle projection (which is considered most acceptable) and 85.9 million on the high projection. South America, which has reached 110.5 million in 1950, will attain 207.4 million, 222.9 million and 239.3 million on the low, middle and high projections respectively.

One of the features of the method is that it provides figures for the population composition by sex and age. In Central America the middle projection shows a decline in the proportion 0–14 years of age from 41.9 per cent in 1950 to 39.6 per cent in 1980, while the proportion 60 years and over rises from 4.2 per cent to 5.4 per cent. The continued high proportion of children creates problems in education and economic development.

The average density of Central America was 14 persons per square km. in 1950 and on the projection will be 29 in 1980. The substantial variations in density which have existed up to now are likely to continue, due as they are to topographic, climatic and historic factors. Thus Mexico, which contains three-quarters of the population of the area shows 28 persons per square km. on the middle projection in 1980 with other countries varying from 5 in British Honduras up to 112 for El Salvador.

Average density of South America will increase in the same period and on the same projection from 6 persons per square km. to 12. The highest figures are 26 for Ecuador and the lowest are 3 for Guiana. Brazil, which has a little more than half the population of South America, shows 12 persons per square km. for 1980, the same as the whole of South America. It is characteristic of the growth of South American population as projected to 1980 that those areas which are already heavily populated continue to grow most rapidly and those of lower density tend to remain where they are. Thus the areas having a density higher than 25 persons per square km. are shown as rising from 42 millions in 1950 to 150 millions in 1980; those on the other hand with density lower than 1 person per square km. grow only from 4 million to 5 million in the same period. Population pressure is more likely on the west coast of the continent than on the east coast, the west coast having much less good agricultural land. On the east coast large populations will be able to expand towards the interior of the continent.

Numerous interesting points are raised in the two monographs on the growth of population. The first monograph, dealing with Central America, develops particularly the aspect of method; the second, dealing with South America, is able to take the method for granted and devotes more space to the possible effects of international migration on the population development of the countries of the continent.

Business Fluctuations and Forecasting. *Carl A. Dauten.* Cincinnati: South-Western Publishing Company, 1954. Pp. viii, 518. $5.75.

LEONARD SILK, *Business Week Magazine*

THIS book's chief purpose is to provide instruction in cycle analysis and forecasting as a basis for business planning. "In determining business policies . . . ," Dauten begins, "it is necessary to analyze fully the possible changes in general business conditions and their effects upon a given industry and business. Any other course may lead to aimless drifting."

But how to avoid aimless drifting by forecasting? "In order to forecast business fluctuations," says Dauten, "it is necessary to understand what they are." So Part I

of his book, which closely follows Wesley C. Mitchell and the National Bureau of Economic Research, tells what business cycles are. Part II, also à la National Bureau, tells how to measure them. Part III gives brief sketches of business fluctuations in the United States from 1783 to Korea, and what allegedly caused them. Part IV superficially describes the cycle theories of Hawtrey, Hayek, Schumpeter, Spiethoff, Hobson, Keynes, Mitchell, and others, and offers the author's eclectic synthesis of cycle theory; his synthesis is more Mitchellian than anything else. The section closes with a layout of proposals for achieving economic stability, including not only fiscal and monetary policies but also Irving Fisher's 100 per cent reserve plan, Benjamin Graham's commodity dollar, Frank Graham's commodity reserve plan, the Townsend Plan, Major Douglas' social credit, etc. "Businessmen could also help achieve economic stability," says the author, "by making careful studies of the demand for their products and by pricing them accordingly."

The ground has now been laid for Part V, Forecasting Business Fluctuations. But Part V has little connection with Parts I through IV. The author simply sets forth some techniques of trend projection, correlation analysis, and opinion sampling as they may be applied to forecasting GNP, the Federal Reserve index of production, the sales of an industry or individual business, and price changes. There is also a brief treatment of the Dow Theory of forecasting the stock market. Dauten completes this section, and his book, with a series of thumbnail sketches of forecasting organizations, including Babson's Business Service, the Econometric Institute, the U. S. Economics Corporation, the American Institute for Economic Research, the National Industrial Conference Board, etc. These sketches are all polite and uncritical.

Methods of Crop Forecasting. *Fred H. Sanderson.* Cambridge, Mass.: Harvard University Press, 1954. Pp. xiv, 259. $5.00.

EARL E. HOUSEMAN, *Agricultural Marketing Service*

METHODS OF CROP FORECASTING is recommended to anyone who has an interest in crop forecasts or the problems of crop forecasting. According to the author, his "chief aim was to subject the recent advances in crop-forecasting methodology to a critical appraisal: to study the relations between the different approaches, to evaluate their relative merits, and thus to arrive at a synoptic view of the various aspects of the crop-forecasting problem." Sanderson has fulfilled those objectives admirably.

In the introduction a general perspective of the crop-forecasting problem is developed. To indicate the prospect for long-range forecasts (a few years in advance), past work on weather and crop cycles is discussed briefly. The author then states, "There is, however, no conclusive evidence that any of the periodicities hitherto suggested are real." Also, many crops evidently pass through short critical periods during which weather conditions have an important bearing on final yield. With reference to such crops the author states that " . . . long range weather forecasts cannot be expected to be sufficiently specific to serve as the basis for forecasting . . . " and, hence, concludes: "For these crops there seems to be no hope of making reliable forecasts prior to the actual occurrence of the critical weather conditions."

The book is limited in scope to methods of forecasting crop yields that do not require forecasts of weather. These methods involve three principal approaches: forecasts from eye observations or judgment evaluations of the growing crop, forecasts from sample measurements of plant characteristics, and forecasts based on measurements of environmental factors, particularly weather.

The book is organized in four parts, besides the introduction, the first part being a summary of some statistical techniques that have relevance to crop forecasting.

Most of one chapter in this part, Design of Sample Surveys, was evidently included because the leading books on sampling had not yet appeared at the time the book was being written. In this chapter, basic concepts of statistics or principles of statistics are not always expressed as explicitly as they should be, even though an effort was being made to balance scientific and operating points of view. For example, as one means of adjusting for bias, when samples are not random, stratification of the sample returns by a correlating variable and weighting were discussed. After pointing out that "Such stratification will have the effect of eliminating any bias in the composition of the y-sample that is associated with x." (y is the characteristic under inquiry) the author states without any further qualification: "Highly selective samples, such as those obtained from voluntary reporters or mailed questionnaires, may be used."

In the second and remaining chapter of Part II, the use of correlation or regression methods to develop a forecasting formula is presented in a manner which brings out the pitfalls, requirements for validity, and the logic involved. Considerable emphasis is given to the fact that many forecasting formulas have been proposed but " . . . most of them are of little value for forecasting purposes because they do not represent tests of *a priori* apotheses, but the result of a systematic selection of the winning combinations of predictors from a large number of possible predictors."

Parts 2 and 3 cover estimating crop acreages and forecasting crop yields from judgment evaluations and plant measurements. The nature of the problems is made clear. Shortcomings of the procedures followed by the crop reporting service and difficulties of building improvements into the present system are also made quite evident. Results of past research in this country and abroad are discussed and adoption of methods that are more objective and less subject to biases is advocated.

In Part 4, Sanderson presents an impressive and realistic picture of the status of knowledge of the way in which environmental factors influence plant growth and yield. Inadequacy of previous records to describe a crop's environment and of knowledge about how environmental factors in combination affect yields are cited as major deterrents to forecasting from observations on a crop's environment.

Productivity Measurement, *Volume I, Concepts.* By the European Productivity Agency, The Organization for European Economic Co-operation. Paris: The O.E.E.C., 1955, Pp. 143. $1.00. Paper.

JOHN W. KENDRICK, *National Bureau of Economic Research*

THE Organization for European Economic Cooperation has, from its inception, been concerned with the advancement of productivity. This aim necessarily involved efforts to define and measure productivity. A preliminary attempt at definition was made in the earlier paper published by the O.E.E.C., "Terminology of Productivity." The present volume was intended as a more advanced, collaborative exploration of concept and measurement. Because of differences among the several authors, the study was published as a collection of essays with an introduction by the editor, G. Duerinck.

In some ways it is too bad that at least part of the volume could not have represented a composite analysis. In various important regards the authors show large agreement, and there is considerable repetition in the various essays. Insofar as the authors place significantly different emphasis on matters of concept, or develop different ramifications of productivity analysis, these could have been handled in separate annexes.

Most of the authors, and the editor, conceive of the term "productivity" as covering the relationship of output to any or all of the associated inputs. They write of a

"wide range of productivity measures": the ratios of various measures of output to various measures of the inputs, collectively or individually.

If productivity indexes are designed to measure changes in the productive efficiency of producing units or complexes, it seems clear that output (gross or net) must be related to a composite of associated inputs (total, or direct factors). The emphasis of the analysis by Walstedt, and to some extent that of Fürst and Ruist, is on "total productivity," as the French term it. Analysis of changes or differences in the individual output-input ratios are desirable supplements in order to identify the areas in which savings of cost elements have been achieved.

It is misleading to the non-specialist, however, for ratios of output to partial inputs to be called "productivity" indexes. So termed, they will inevitably be interpreted either as measures of efficiency of the particular factor, or of productive efficiency generally, although they are measures of neither. Fürst criticizes the U. S. Bureau of Labor Statistics on this score for its leading role in making current the expression "labor productivity."

The editor and Rostas and Siegel seem to believe that there is a particular virtue in measuring output in relation to labor input. The practical reasons are given that labor input may be measured readily in terms of persons or man-hours employed, while capital input presents greater difficulties of quantification; and in any case labor is the chief input. The latter point was more convincing in Ricardo's day than at present; and for many individual industries it is not the case. The former point is a challenge to economic statisticians which has begun to be met in recent studies of wealth and capital coefficients. Siegel sees the output per man-hour measure as particularly relevant for the "very long-run." It would have to be a very long run indeed before capital becomes so abundant relative to labor as to be costless—if this is what he has in mind.

Rostas quotes an earlier statement by Siegel that the output per man-hour measure "is particularly appropriate for a society in which, in Marshall's words, man is both 'the end and an agent of production.' " Rostas concludes: "It is through this dual role that productivity of labour becomes an important indicator of the standard of living." But "labor productivity" is really not a satisfactory measure of either productive efficiency or planes of living. Marshall himself conceived of increasing return as a relationship between output and labor plus capital inputs. Changes in the ratio of labor input to population make the labor productivity index a faulty indicator of planes of living; substitution of capital for labor, as Ruist points out, creates an upward bias in output per man-hour as an efficiency measure. Walstedt rightly maintains that the increase of output relative to labor time expended is not, *per se*, a target of industrial action; rather, a net saving in all cost elements indicates improved industrial efficiency.

The French working party proposes two synthetic types of measures of dubious value. In order to get at the "integral productivity of labour," they suggest dividing all costs by an average wage-rate. This is reminiscent of Adam Smith's "labor commanded" approach to value. It is deficient because non-labor cost so deflated need not yield a true measure of other factor input. Insofar as productivity has increased in the production of materials and capital goods, prices of these would fall relative to wage-rates; thus, "labor equivalent input" would have a downward bias, and the productivity measure an upward bias—although this might be offset (or augmented) by changes in interest rates.

The other questionable proposal is to subtract the constant-value sum of non-labor inputs from real output, and relate the residual to man-hours in order to obtain a measure of "net labor productivity." This is statistically possible, but it seems in-

consistent to compare an output measure which reflects savings in *all* costs with labor input alone. It is more defensible to compare real net industry output (excluding purchased goods only) with total factor input. These measures are parallel, and consistent with productivity indexes for the economy as a whole as pointed out by Fürst in his section on the use of census data in productivity analyses.

The French working party contributes a helpful annex giving numerical illustrations of their various concepts. The effect of using unit values weights from each of the two periods being compared is illustrated. They show quantities of labor input falling relative to other inputs, while the relative price of labor rises. Thus, use of period II weights yields a smaller increase in composite input than use of period I weights. Conversely, of the two types of output, one shows relative increases in both production and price; thus II weights result in a larger increase in total output than I weights—and the index number effect on changes in the productivity ratio is accentuated. Actually, it would have been more realistic (as far as American experience goes) to assume a negative correlation between relative changes in output and prices; in this case, the index number effect is mitigated in the productivity ratio.

Walstedt presents several methods for making international comparisons. He recommends the method of weighting the inputs and outputs of the producing unit or complex in each country by the unit costs or prices prevailing in one of the countries. He demonstrates the situations in which alternative types of comparisons, such as manhours per unit of output, may give good, or poor, approximations to the differences in productive efficiency as revealed by total productivity comparisons.

This brief review of a few major points cannot do justice to a volume in which many of the subleties of productivity measurement are treated. Siegel, in particular, exhibits a generally high degree of sophistication in his discussion of technical matters. Mention should also be made of Ruist's treatment of the relationship between productivity and wages. He considers the implications of a policy which ties changes in wage-rates to changes in output per man-hour, with special regard to its effect on the unit compensation of nonlabor factors.

One ends the volume with the feeling that the quotation from Sullivan by Siegel applies: " . . . it is much easier to make measurements than to know exactly what you are measuring." The authors do not go behind the changes or differences in the productivity measures even to the extent of identifying the basic forces at work, such as technological change, rates of output relative to capacity, economies of scale, and the contribution of intangible capital. The meanings of the use of alternative conventions, as regards weighting for example, are not explored. And despite the attention devoted to capital input and total productivity, questions as to the concept and measurement of real capital input are largely by-passed.

It is to be hoped that further clarification of these and other matters will be found in the second and third volumes of the series. These are planned to cover problems met and results obtained in productivity measurement at the levels of the firm and nation.

Income of the American People. *Herman P. Miller.* New York: John Wiley and Sons, 1955. Pp. xvi, 206. $5.50.

PETER O. STEINER, *University of California (Berkeley)*

THIS able contribution to the census monograph series analyzes Census data on the size distribution of personal income with an eye to identifying the economic and sociological factors associated with the dispersion in income. Beginning with an examination of the over-all income distribution it subsequently treats geographic location, color, occupation, age, sex and family status as classifying variables, used

singly and in combination. Attention is divided between a cross sectional analysis of the data for 1950 and an analysis of changes in the degree of income dispersion between the late thirties and 1950. The monograph concludes with an evaluation of the quality of census income data, most of which appeared previously in this journal.[1]

Miller argues that the extreme skewness found in the over-all size distribution of income is largely due to the merging of heterogeneous subgroups of income receivers whose income distributions are, in general, nearly symmetrical. Occupation (along with its correlates) emerges as the key variable and is subsequently subjected to a detailed analysis that, in my view, is the major contribution of the book. But other variables are not neglected, and short chapters concerning geographic location and color, age, and family status add to our understanding of the size distribution of income. Space limitation prohibits a comprehensive summary of the findings, but a brief sample is presented below.

The analysis of geographic location develops a number of net relationships with income. The well-known positive association of income and size of place is largely, but not entirely, identified with differences in occupational opportunities. Income differences of whites and non-whites are substantial even when factors such as age, sex, education and size of place are taken into account, and this relationship helps explain the systematic regional difference of the South from the rest of the nation. However a regional disadvantage of the South remains (net of color, sex, residence, and community size) which is not particularly related to the occupational structure, and which must be explained in terms of wage-price relationships, differences in productivity and so on.

The analysis of occupation, both for major occupational groups and for a detailed list of specific occupations (118 for men, 32 for women) reveals the nature of differences in the level of income (which are highly stable over time) and illuminates the decrease in the dispersion of income between 1940 and 1950. The latter is accounted for by the relative gain in level of income of the occupations which typically are located at the low end of the income scale, and by the decrease in the within-occupation dispersion in the great bulk of the specific occupations.

There is, of course, a great deal more to the book than is summarized above. However, since this volume should become mandatory reading for anyone at all interested in the income structure of the American people, it seems unnecessary to summarize further.

Miller has deliberately chosen to highlight his major findings at the possible cost of some oversimplification of the interpretation of subtle and complex relationships. He has not hesitated at a number of points in going well beyond his data in hypothesizing about possible explanations of underlying causes. The benefit of this procedure is to provide the reader with many insights; the cost is that each reader will disagree with both interpretation and causal explanation in some instances. I will not catalog my disagreements because of the feeling that statisticians—particularly at the Bureau of the Census—should not be encouraged to play it safe and thereby produce less stimulating analyses.

A reviewer, however, is entitled to one quibble. The least satisfying of the major chapters concerns the "explanation" of the skewness of the over-all income curve in terms of a series of more or less symmetrically distributed subgroups. The apparent symmetry of some of these seems fairly synthetic, in terms of the heterogeneity within subgroups with respect to the nature of employment (part-time—full-time, for example) and other factors. After all, the existence of dispersion suggests heterogeneity with respect to *some* other variable. Further, whatever symmetry is in fact present

[1] Herman P. Miller, "An appraisal of the 1950 census income data," *Journal of the American Statistical Association*, vol. 48, March 1953.

is visually exaggerated in the charts by the extrapolations of the frequency polygons into the negative income ranges which inexplicably overrepresent the reported frequency densities.

The only major disappointment I have is that while the analysis is multivariate in spirit, the major analytic device used is the cross classification table. While in general this technique is sufficient unto the data, there are places where a more formal analysis of variance would have helped and where appropriate techniques might have identified the specific contributions of individual factors to an over-all gross relationship. Frequently one suspects that a formal analysis had been made but was suppressed in the interest of widening the market for the book. Perhaps statisticians should devote some of their efforts to educating editors.

But let no amount of criticism detract from the main evaluation. This is an outstandingly useful piece of work, and represented, for the reviewer, an auspicious introduction to the census monograph series.

Report on the Sample Survey of Ceylon's Consumer Finances. *B. B. Das Gupta,* Director of Economic Research. Colombo: Central Bank of Ceylon, 1954. Pp. ix, 34, +40 of Tables. 50 cents or 316d. Paper.

IRVING SCHWEIGER, *University of Chicago*

THE power and economy of modern sampling methods have facilitated investigations of the distributional structure of aggregate consumer income and expenditures in a number of countries. This report for Ceylon for the period April 1952 to March 1953 is one of the most ambitious studies undertaken so far. In the course of three sessions, interviewers obtained data from individual households on labor force, income, detailed expenditures, saving, accumulated savings, anticipated changes in spending and saving if income increased or decreased by 10 per cent, sanitation facilities, and other matters. The investigation provided many figures previously unavailable or not available in correlated form.

Estimation of the value of these data is not simple. The directness and apparent simplicity of the field survey technique conceals a number of subtle problems of communication. Errors in reporting of information are pervasive and can be very substantial. For example, in income investigations in the United States, rechecks of identical consumer units after one month found that only one-half of consumers reported the same broad income bracket both times. The data presented in the Ceylon survey suggest that there were difficulties in obtaining full, accurate, and reliable reports from consumers for many of the items covered. A less ambitious questionnaire may have had fewer difficulties with the smaller number of areas covered.

National Incomes and International Trade. *Hans Neisser and Franco Modigliani.* Urbana: University of Illinois Press, 1953. Pp. xviii +396. $7.50.

ARNOLD C. HARBERGER, *University of Chicago*

THIS book, prepared under the auspices of the Institute for World Affairs of the New School for Social Research, is an empirical investigation into the relationships between fluctuations in national incomes and fluctuations in the components of world trade in the interwar period. Its core is a system of estimated demand functions for the imports and exports of six countries or groups of countries. Separate functions were estimated for imports or exports of raw materials, foodstuffs, and manufactured goods.

Together with Polak's recent work, the book represents an important contribution to our understanding of the mechanism by which income fluctuations were trans-

mitted from country to country during the interwar period. The fact that it does not tell the whole story should be neither a surprise nor a cause for complaint. One should note, however, at least three respects in which a clearer picture might be desired.

First, export and import demand functions are estimated for only four countries: the United States, the United Kingdom, Germany, and France. The remaining countries are lumped together into two large aggregates, according to whether their exports are predominantly manufactured or primary products. Readers interested in the economies of countries other than the four mentioned above will accordingly be disappointed, even in the cases of those countries (Canada, Australia, the Netherlands, for example) whose data would have permitted the estimation of separate functions.

Second, the estimates of country i's marginal propensity to import from country j are synthetic, in the sense that they were not directly derived from the data. In no case were j's exports to i correlated directly with i's income. In some cases j's exports to i of a given class of goods were correlated with i's total imports; here i's marginal propensity to import from j was derived by multiplying i's estimated over-all marginal propensity to import that class of goods by j's marginal share in i's total imports of the class. In other cases j's exports of a given class of goods were correlated with the (weighted or unweighted) sum of other countries' imports of that class; here i's marginal propensity to import from j was taken as i's estimated marginal propensity to import the given class of goods times j's marginal share in the aggregate of other countries' imports of the class. Though Neisser and Modigliani's procedure is somewhat more sophisticated than Polak's (in which j's exports are simply correlated with an index of the level of world trade), it cannot be relied upon to yield very accurate estimates of marginal propensities of given countries to import from particular sources.

Third, the estimates of price elasticities of import and export demand are probably unreliable, and are so recognized by the authors. Although relative price variables were experimentally tried in all functions fitted by the authors, their coefficients turned out to be significantly different from zero in only three out of 19 import functions and in only 10 out of 37 export functions, and even in these cases the elasticities were considerably lower in absolute value than those emerging from recent studies which concentrated on the estimation of price elasticities.

The material is presented in workmanlike fashion, so that the reader can follow the estimation process from data through estimating procedure to result. In addition there are useful discussions of both the theoretical and the statistical problems that confronted the authors. One should mention also an extremely interesting chapter in which the actual levels of trade in the 1930's are compared with the levels which would have been predicted on the basis of "normal" relationships between the components of trade and industrial production. The inference is drawn that the increase in trade restrictions among the industrial countries after 1932 was responsible for a significant reduction in the level of world trade.

This is not a book for the amateur or casual reader, but it should be on the bookshelf of every expert in the field of international trade.

The Terms of Trade—A European Case Study. *Charles P. Kindleberger.* New York: The Technology Press of Massachusetts Institute of Technology and John Wiley & Sons, Inc., 1956. Pp. xx, 382. $9.00.

STEPHEN ENKE, *The RAND Corporation*

THE terms of trade have long been a matter of public and professional concern in many European countries (particularly the U. K.) and certain primary producing countries (e.g., Chile, Australia, and Malaya). Kindleberger, helped financially by

the Merrill Foundation, has made an empirical examination of the terms of trade of eight European countries versus the rest of the world from 1870 to 1952. He has also analyzed his resultant indices for possible confirmation or refutation of a number of theories concerning the effect of commercial policies, exchange rate changes, foreign and domestic investment, business cycles, changes in demand and supply for products, and economic development, upon the terms of trade of "industrial" Europe.

Kindleberger properly explains the limited significance of the terms of trade concept at the outset. American readers should know that the terms of trade of a nation with, say, the rest of the world has much in common with the concept of parity price percentages for farm products that has so long bedevilled U. S. politics; for example, the terms of trade are said to have gone against the U.K. if the prices of its exports fall relative to the prices of its imports. In Europe it is popularly believed that world prosperity raises commodity prices more than manufactured goods prices, so that the terms of trade for Europe, which typically imports commodities and exports manufactures, is worsened. This example itself suggests that the terms of trade concept is a somewhat limited, if not spurious, measure of economic welfare, as it ignores quantities exchanged and production costs.

The Annual Index is one of the two main time series indices constructed by Kindleberger. His "industrial" Europe includes the United Kingdom, Germany, France, Italy, the Netherlands, Belgium, Sweden, and Switzerland. For Sweden, Switzerland, the Netherlands, and Italy, Kindleberger could not find satisfactory data or series prior to this century. The Schlote series was used for the United Kingdom and that of Soltau for Germany.

The logical development of the Annual Index is as follows. With 1913 as a base, a merchandise terms of trade index series was developed for each of the eight countries, from the earliest possible date, with adjustments after 1900 designed to eliminate the effect of international trade among the eight countries. This merchandise terms-of-trade index for each country is the ratio of its export unit-value index to its import unit-value index. There is no correction for the import content of exports, nor is it clear that this would be desirable given the purpose of the investigation. The merchandise terms of trade indices of the eight European countries were aggregated in accordance with their annual quantities of trade expressed in 1913 dollars. Attempts were made to adjust these series to include foreign earnings from shipping and capital loans, thereby deriving a current account index. To some extent these steps are described in Appendix A.

As the initial elements are the export and import unit-value series, by years and countries, something should be said about a few of the conceptual and practical difficulties that exist. Ideally, an export unit value series, for instance, should show annual changes in the "value" of a "unit" of exports. Over an 80 year period what is a representative "unit" of exports? All the usual implications of using base year weights or current year weights apply here; for instance, if prices rise, and this is because of demand increases, quantities probably increase, too, and base year quantity weights will understate the effect. In practice, quantity weights for specific commodities may not be employed at all; instead, the value of exports, by categories, may be deflated by some crude quantity unit of measure (such as tons weight in the case of iron and steel manufactures). For international trade there is also the problem of whether export and import values should or should not include freight charges. It is customary to value exports f.o.b. and imports c.i.f., but this is invalid here unless foreigners always collect their purchases and deliver their sales. Another problem is encountered when aggregating indices for individual nations. In this case weights were based on the dollar value (in 1913 dollars) of each nation's annual trade: it must frequently have

been difficult to know what dollar exchange rates to use in the case of countries (e.g., Nazi Germany) having exchange control, and in effect many different currency exchange rates, during certain periods.

The Detailed Index was constructed for six selected years, for nine commodity categories, for each of the eight European countries, and separately showing its export and import unit-value index for trade with the other industrial European countries, other non-industrial European countries, the United States, areas of recent settlement (e.g., Argentina), and all other areas. This laborious task was undertaken in the hope, among other things, of explaining historical trends in the terms of trade of underdeveloped and primarily commodity producing countries with industrial Europe. The statistical operations involved are described in Appendix B.

So far as one can tell, Kindleberger has made the best of a difficult and tedious statistical job. He does not gloss over the conceptual and data inadequacies. He would be the first to admit that any year-to-year change of only a few per cent in the current account index of a single country may have no significance.

The Annual Index and the Detailed Index were constructed with the hope that they might confirm or refute a number of economic theories concerning the terms of trade. For instance, Kindleberger uses his indexes to evaluate the Carl Major Wright thesis, Engel's law, the "demonstration effect" argument, and many other widely held views. It appears however that in every case the indices *per se* throw little if any light on these subjects. The data neither support nor deny.

Paradoxically, this book, although ostensibly an empirical study, should be bought and read for its qualitative contents. Kindleberger has done an excellent job of surveying, in a way that should be comprehensible to that "intelligent layman," a great many theories concerning the ways in which the economic welfare of a nation rises or falls relative to that of others. And, while he is explaining why his Annual and Detailed indexes have little to say about these theories, he incidentally describes in a sensible way what events in international economic affairs were really important at certain times in the lives of various countries. There is all sorts of incidental economic intelligence, too, as that the Sheik of Kuwait cannot spend his oil royalties fast enough (p. 205), or that Javanese sugar cane (unlike Cuban) must be planted 20 months in advance of each harvest (p. 269). Thus the reader who feels empty after the main empirical course can fill up on a profusion of factual canapes throughout the meal.

One of the author's most important "conclusions"—with which this reviewer concurs subjectively—has little to do with the summary terms of trade indexes. This *obiter dictum* is that the countries with improving terms of trade are benefiting sometimes from luck (they have resources that are becoming more important to the world), but also from starting new industries quickly and permitting resources to leave older industries suffering from failing demand or falling prices. Kindleberger sees economic progress stemming from a combination of adequate capital, an intelligent working force, and entrepreneurial energy. The developed countries are improving their terms of trade versus the undeveloped countries, not because the former produce industrial rather than primary products, but because the latter lack the three requisites listed above.

Kindleberger has written an interesting book in a lucid style. His scholarship is evidenced by many bibliographical references and wide review of theorems concerning the terms of trade. His knowledge of international economic life is considerable. The indexes that he has constructed testify to his industry and perseverance, and their somewhat negative implications should still have some value if they prevent a too eager acceptance of some popular views regarding the determinants of the terms of trade.

Capital Formation in Canada 1896–1930. *Kenneth Buckley.* Canadian Studies in Economics, No. 2. Toronto: University of Toronto Press, 1955. Pp. x, 163. $3.25.

ERIC SCHIFF, *Machinery and Allied Products Institute*

THIS book offers a careful descriptive and statistical analysis of real investment in Canada during the decades before the great depression. The study is divided into eleven chapters, of which the first six and the last five form two major sections. The first chapter outlines a concept of real capital formation broadly similar to that underlying the leading American studies in the field (Kuznets; U. S. Department of Commerce). Expenditures on resource development are included only to the extent that they involve outlays on structures or machinery and equipment. Chaps. Two to Six offer an historical account of capital formation in the main lines of real investment during the period surveyed. The second chapter deals with investment in the wheat economy, the backbone of Canadian economic development in the period.

Chap. Three discusses investment in transportation, which during the period studied means, by and large, investment in railroads. While railroad building has of course transformed the economies of all countries where it took place on a large scale, there may not be many countries where the transformation was as fundamental as in Canada. As Buckley emphasizes, it was the railroads which changed Canada from a maritime economy based on timber and fish, to a transcontinental economy based on wheat. Recognition of this historical key role of railroad building does not prevent the author from being critical of its extent during part of the period. He thinks that at times the extensive as well as the intensive investment in railroads was seriously overdone.

In the following chapter on urban building the author was able, on the basis of building permit data reaching back to 1866, to examine the relation between building cycles and population movements over an unusually long period. In Canada during these decades, population movements were to a large extent reflected in fluctuations in the rate of immigration. Buckley's analysis shows that both this rate and the series of residential building permits fluctuated in long 15- to 20-year cycles, the immigration series nearly always leading. While the general picture is what might be expected, the closeness of the (lagged) parallelism of the two curves portrayed in Fig. 6 on p. 41 is impressive. A chapter on public investment, and one on savings and capital flows, both of them highly informative, bring this first main section of the book to its conclusion.

The last five chapters present an account of the sources and procedures used in measuring real investment. For construction, it was necessary to combine estimates of four component series (value of construction materials used, wages and salaries paid to construction workers, overhead expenses, profits) which in turn had to be derived on the basis of fragmentary data by techniques involving some admittedly arbitrary assumptions.

In deriving the estimates of investment in machinery and equipment, the commodity flow approach was in most cases at the basis of the procedure. The method paralleled in this respect that used by Kuznets, Shaw, and the U. S. Department of Commerce in estimating investment in producers' durable goods in this country. As in all these cases, interpolation for intercensal years, and derivation of estimates for trade and distribution markups (in order to pass from values at producers' prices to values at prices paid by final users) were the two thorniest problems. Their discussion in Buckley's study bespeaks great care in striving for the best solutions possible with the available source material.

Investment and disinvestment in inventories is defined, again in agreement with

the American studies, as additions to or subtractions from the physical stock, valued at current prices. So the problem of how to derive acceptable adjustments of the book valuation of inventories occupies a major part of this chapter.

The study has two self-imposed limitations. For one thing, it is almost wholly confined to the analysis of gross investment. In the introductory chapter the author states that for the few occasions where it was found desirable to give estimates of net investment, normal depreciation and obsolescence allowances as charged under customary business practice were considered acceptable approximations to capital consumption and replacement demand. For the quinquennial estimates of net private domestic investment from 1900 to 1930 as tabulated in Chap. Six, 10% of average gross national product was taken as a reasonable approximation to average replacements. Admittedly, neither of these two ways of estimating capital consumption yields any but extremely crude results, and further research will be needed to obtain an adequate picture of this angle of Canadian capital formation. The second limitation lies in the "cutoff" date, 1930. The reader inevitably wonders how the described historical conditions or statistical series developed further under the post-1930 impact of depression and war.

Since, however, both of these limitations were in the plan of the study, they cannot be called shortcomings. Within its self-imposed scope, Buckley's analysis is highly rewarding. A rich source material, and a considerable degree of refinement and circumspection in applying statistical techniques to it, has gone into the relatively narrow space of 163 pages. The presentation is lucid, and the critical mind with which the author reviews a number of important historical developments contributes to make the study interesting reading.

The Ownership of Tax-exempt Securities, 1913–1953. Occasional Paper 47. *George E. Lent.* New York: National Bureau of Economic Research, 1955. Pp. x, 140. $1.50. Paper.

KENNETH M. WRIGHT, *Federal Reserve Bank of New York*

STUDENTS of public finance and of the capital markets will find this a useful book because of its statistical tabulations, its descriptive material, and its stimulating analysis of the tax considerations affecting the investment decisions of various investing groups. Although tax-exempt securities outstanding at the present time consist largely of the issues of states and political subdivisions, prior to World War II the wholly- or partially-exempt securities of the Federal Government and of various Federal credit agencies were also of considerable importance. Lent has carefully recorded the amounts of each type of tax-exempt security outstanding as of June 30 of each year from 1913 to 1953, together with the holdings of major investing groups.

The statistics of the ownership of tax-exempt securities have been compiled partly on the basis of existing data and partly through the preparation of new estimates for the holdings of fire, marine, and casualty insurance companies, life insurance companies, taxable corporations, fraternal societies and other tax-exempt institutions. Individual holdings are taken as a residual derived from the estimates of holdings by all other groups. Since data for the amounts outstanding are most readily available for the end of fiscal-year periods, Lent has presented the statistics of ownership as of June 30 dates, often through interpolation of available calendar-year data. However, some readers will find it awkward for many purposes to compare these June 30 statistics with other data. Although there may have been specific reasons for using June 30 dates, a more desirable alternative might have been to interpolate fiscal-year data on amounts outstanding to a calendar-year basis, and then compare directly to ownership statistics which are often available on that basis.

Within this statistical framework, Lent outlines in Chap. 2 the principal legislative developments and judicial decisions with respect to the flotation of tax-exempts by state and local governments, the Federal Farm Loan System, and the Federal Government itself. The chapters that follow are concerned with the ownership of all classes of tax-exempt issues by governmental bodies, institutional investors, and individuals.

The varying tax incentives which affect the different investors in tax-exempt securities are the principal factor examined by Lent in his explanation of the shifts in ownership of tax-free securities. Since the advantage of owning such securities derives principally from the fact that the income is exempt from federal taxation (and in some cases from state and local taxes), the attraction of different investor groups to tax-exempt securities varies according to the relative impact of federal taxes upon them. Accordingly, those subject to higher federal tax rates tend to bid away these securities from those who find in them a lower tax advantage. Much of Lent's analysis is built upon this one aspect of the ownership of tax-exempt securities.

A decline in the interest of state and local government sinking and trust funds in state and municipal bonds has occurred since 1940 because these investors are themselves exempt from federal taxation. With a higher yield available on fully taxable federal securities than on municipal bonds, the assets of these funds have been switched increasingly to federal securities (p. 44 ff.).

In Chap. 4, tax considerations are also used to explain changes in the ownership of tax-exempt bonds by institutional investors such as commercial banks, life insurance companies, fire, marine and casualty insurance companies, and taxable corporations. In examining this segment of the market, Lent finds that "The relative interest differential between high-grade municipals and industrial bonds tended to vary with corporation income tax rates" (p. 63). It should be remembered, however, that individual investors have historically constituted an important share of the municipal market, and that the differential in yields also reflects variations in personal tax rates. As Lent points out in his preface, the "critical factor" determining the size of the interest differential between taxable and tax-exempt securities "has been the level of the corporation tax rate and its relationship to the graduated income tax schedule" (p. 4). Unfortunately, no attempt has been made to present an integrated analysis of the relative or combined effects of the corporate *and* personal tax rate on the size of the interest differential.

Lent's treatment of the role of individual investors in the tax-exempt market (Chap. 5) is put mainly in terms of the breadth of the "individual market," which is determined by the number of taxpayers whose personal tax rates exceed the corporate tax rate, thus making such securities of more value to them than to taxable corporations such as commercial banks. In 1952, for example, individuals with incomes of $29,000 and over "were equated with the corporation rate" (p. 87) and thus found it worth-while to bid against taxable corporations for tax-free securities. This approach can be carried only to a certain point, however, as Lent is careful to point out from the experience of the 'thirties, when the share of individuals in the ownership in tax-exempts gave way to the "institutional market fortified by excess reserves of commercial banks and the plethora of loanable funds in the hands of life insurance companies" (p. 86).

Another dimension is added to Lent's analysis of the "individual market" by the consideration of alternative investment opportunities, particularly corporate bonds and stocks. Lent sets forth the basic principle as follows:

> At a given level of income tax rates, there is a point on the income scale where the tax rate paid on the next dollar of income is equivalent to the inter-

> est differential between tax-free and taxable investments of similar characteristics. Below this point the purchaser of tax-exempt securities would sacrifice more in interest than he would gain by exemption; above it the advantage of buying tax-exempts increases with the marginal tax rates (p. 89).

He then presents charts for various time periods showing the income levels at which there existed a tax incentive to purchase tax-exempt bonds rather than corporate bonds. For example, married persons with incomes of $25,000 in the 1932–1941 period found the yields on high-grade municipal bonds and on corporate bonds equally attractive; however, the postwar income-splitting provision raised this "critical" income level substantially.

Lent's analysis of some of the tax incentives affecting the ownership of tax-exempts by different investors cannot be considered as a complete explanation of the many factors at work in this market. Tax incentives are doubtless of strategic importance in many situations, but a rounding-out of the analysis requires an examination of the supply of tax-exempt securities as compared to alternative investment opportunities (including taxable securities and business properties), measured against the volume of available funds in the hands of potential investors—in short, a fuller statement of the sources and uses of funds in the capital market, of which the market for tax-exempt securities is only a part.

It is only fair to state that Lent himself appreciates the deficiencies of an exclusively tax-oriented approach to the ownership problem. As he states in the preface (p. 4), "The height and degree of graduation in income tax rates, the comparative supply of municipal and various competing types of securities, the relative attractiveness of safety and speculative gain, the requirements of various types of investors, the credit policies of federal agencies, and other factors may all play a part [in the interest differential between tax-exempt and taxable securities]."

Fiscal Policy, Its Techniques and Institutional Setting. *James A. Maxwell.* New York: Henry Holt and Co., 1955. Pp. vi, 218. $2.90.

M. H. Miller, *Carnegie Institute of Technology*

This well-written little book is essentially a summary of the thinking of economists during the last 10 years on the subject of countercyclical fiscal policy. By 1946 or 1947, a substantial consensus had been reached within the profession with respect to the underlying theoretical analysis of the problem of economic stabilization. Thereafter, research in the field of public finance was increasingly focused on the institutional obstacles to effective implementation of fiscal policy. To keep fluctuations in employment and prices within tolerable limits, the Federal government must stand prepared to vary the level of its own expenditures and taxes—increasing its net contribution to the income stream when private employment falls and draining off incomes as inflationary pressures develop. Much of current government spending, however, is of a type where considerations of efficiency or urgent continuing needs for the services rule out alteration in program levels to meet temporary changes in economic conditions. The required adjustments in those programs which do contain some margin of discretionary flexibility must each separately run the gantlet of a legislative process which, however admirable it may be for serving other important objectives in a democratic society, creates vast opportunities for delay and inconsistency. When account is also taken of the still very primitive state of economic forecasting, it is clear that success in getting the necessary changes before extensive damage has been done cannot be taken for granted.

Maxwell believes it unlikely (and probably undesirable) that better control over the direction and timing of fiscal policy will be achieved by a delegation of much of the traditional power of Congress over the finances to the Executive or by a commitment by Congress to some sort of formula-timing scheme. Fortunately, however, as his detailed survey of existing expenditure programs and taxes shows, the budget already contains a considerable degree of automatic or built-in flexibility, as well as several areas in which this flexibility could be significantly increased without creating a threat to Congressional prerogatives. This tendency of the budget to adjust itself automatically in response to changes in the level of national income can materially reduce the amplitude of business cycles as well as provide the managers of fiscal policy with some of that safety margin in the timing of changes made necessary by the inadequacies of forecasting. But built-in flexibility cannot do the job of economic stabilization by itself. New spending and tax legislation will frequently be required and without delay. Maxwell's reading of past history and the current political consensus, however, leads him to the optimistic conclusion that Congress can and will respond rapidly enough to vigorous leadership in the executive branch.

To those who have some familiarity with the recent literature on public finance and fiscal policy, much of the most interesting sections of Maxwell's survey of the budget components are likely to be the chapters on the expenditure programs. They should serve among other things, to alert economists and policy makers to the dangerously thin margin of flexibility in the current social security program as well as to temper current negative attitudes towards agricultural price supports. The discussion of taxes, on the other hand, is carried out on the whole at a higher level of generality and provides fewer new insights even for the nonspecialist. The analysis of built-in tax flexibility, moreover, seems to be in error, or at least misleading at a number of points. Examples are the introduction of the partial equilibrium concept of elasticity of demand in the discussion of excise taxes; and the failure to point out, in connection with the built-in flexibility of the corporation income tax, that corporate profits themselves are a part of the total built-in flexibility of the economy.

Although the book does accurately and lucidly summarize much recent thinking with respect to fiscal policy, specialists may question whether the present is really a profitable time in which to attempt such a summary and assessment. Profound and still imperfectly understood changes have recently been taking place in the structure of the economy—e.g., in consumer spending habits, in business planning and budgeting practices, and in wage structures and methods of wage payment. New and important empirical tools for the analysis of the structure of the economy, such as input-output models and money-flows accounts to name just two, have only recently become available. Sharper theoretical models at both the macro and micro levels are known to be now under development and our knowledge of control systems generally is expanding rapidly. The field of fiscal policy, in short, seems likely to undergo some fairly drastic revision in the not too distant future.

Pricing and Price Differentials on Over-the-Counter Markets. *Irwin Friend, Morris Hamburg, and Stanley Schor.* Philadelphia: University of Pennsylvania Press, 1955. Pp. vi, 121. $1.25. Paper.

PETER L. BERNSTEIN, *Bernstein-Macaulay, Inc.*

THIS book contains a rich treasure of statistical information about security transactions in over-the-counter markets in 1949 and, less completely, in 1951–52. The coverage is impressive, including such features as an analysis of actual prices of

transactions relative to bid-and-asked quotations in dealer and public markets and an extremely detailed breakdown and comparison of the costs of security transactions over the counter and on the New York Stock Exchange. Most of the data, furthermore, are broken down by size of transaction, size of issue, size of dealer, geographical location of dealer, type of investor, and type of security.

There is more here, however, than meets the eye, and it would be interesting to know how much of it met the eyes of the authors. For the study, if presented in more lively fashion (and there is little that is lively about this presentation), could be a highly useful handbook to guide the uninitiated through the intricacies and pitfalls of over-the-counter transactions. Here are just a few facts which the investor could use to good advantage:

(1) The investor almost never knows whether, at the same moment, other investors are paying less for a security or selling for a higher price than he is, nor will he know what the dealer's spread is if the dealer acts as principal instead of agent.

(2) When selling, he is likely to do so at a price significantly closer to the price at which the dealer will be able to dispose of the security than, when buying, he is likely to buy close to what the dealer has to pay for the security. In other words, customers' sales are made much closer to the "inside market" than are customers' purchases.

(3) When selling a security, the investor is likely to get a better price if the dealer acts as principal rather than agent; when buying, he is likely to do better to let the dealer act as agent and simply earn a commission.

(4) Smaller dealers have larger percentage gross profit margins on each transaction than do larger dealers. This may be because they do more business in stocks and less in bonds or because they have fewer institutional customers and therefore smaller average transactions. But could it not also be the result of dealing with a less sophisticated and less informed group of investors? It would be interesting to know also how closely this difference is correlated with the marked geographical differences in gross profit margins, with price spreads very much narrower in New York than anywhere else.

(5) Dealers take a much bigger mark-up on individual investors than on institutional ones, even when individual transactions run up over $10,000.

(6) What the authors call the "average customer cost" (i.e., the difference between the price that one customer receives when disposing of a security and the price that another public customer pays for that security at the same time) is 5.2% on over-the-counter transactions in unlisted issues and 4.4% on listed issues, compared with only 2.1% on transactions on the New York Stock Exchange.

Thus, there is no doubt that investing or trading in over-the-counter securities is a relatively expensive affair, and, for the uninitiated, a little like shooting in the dark. The authors suggest (but make no effort to prove) that the higher cost may be due to differences in services rendered and that the over-the-counter market may still offer greater security values than the listed exchanges. But that is really the crucial question. The study, for all of its statistical material, misses having real value because it fails to come to grips with this very point.

The factors listed above hint at the possibility that the greater customer cost in the over-the-counter market may be due at least in part to the fact that so large a part of each transaction is hidden from public view and opportunities for discrimination among customers are large. The authors would have performed a real service by exploring the degree to which this is so, for the situation is fundamentally different on the New York Stock Exchange. In particular, the authors might have used the results of their statistical analysis to suggest changes in methods of reporting quota-

tions or prices at which transactions occur in order to provide buyers and sellers with that more complete knowledge which any perfect market requires.

Measurement and Evaluation in Psychology and Education. *Robert L. Thorndike and Elizabeth Hagen.* New York: John Wiley and Sons, Inc., 1955. Pp. vii, 575. $5.50.

R. F. JARRETT, *University of California (Berkeley)*

THIS is a textbook written for future test *users*—teachers, counselors, personnel technicians, etc.—not for theorists. It presents at a very elementary level a great variety of topics with which the neophyte in this field must early make himself familiar. It will have little interest for the statistician *qua* statistician, and its review in this journal must therefore be brief.

The book is carefully organized with an eye to the needs of the person who is just beginning to consider the problems involved in obtaining the variety of measures required for the psychological characterization of persons. The authors discuss, among many others, such topics as the reasons for testing, the assets and liabilities of essay and objective tests, how to write good items, the test instructions, elementary considerations of reliability and validity, methods of reporting test results, etc. Tests of all functional varieties are discussed, among them achievement tests, diagnostic educational tests, aptitude tests, attitude tests, interest inventories, etc. Appendixes contain information on selected commercially available tests of various types. One chapter on "Elementary Statistical Concepts" treats just that, covering in 28 pages elementary descriptive statistics, standard scores, and correlation.

Considerable care seems to have been spent in the writing for, although I have not troubled to estimate the Flesch count, I am willing to wager that college sophomores will find it easy to read and to understand: but more, I suspect they will find it exciting!

The publishers are to be commended for the design and execution of the book; to the layman's eye the typography is excellent and the binding tasteful and adequate.

Proceedings 1955 Invitational Conference on Testing Problems. Princeton: Educational Testing Service, 1956. Pp. 152. $1.00. Paper.

LOUIS GUTTMAN, *Center for Advanced Study in the Behavioral Sciences*

PAPERS presented at a conference sponsored by Educational Testing Service make up these *Proceedings*. They fall into three sections: "Multi-factor ability test batteries in counseling and guidance," "Communication of test information," and "Clinical vs. actuarial prediction."

Domination of the Thurstone approach to the factor analysis of mental abilities is revealed throughout the first section. Almost all these papers indicate that the use of a "general intelligence" score has been largely abandoned in favor of differential scoring on "primary abilities" as revealed by factor analytic studies, whether in the field of mental or mechanical abilities. The opening paper does point out "that when school teachers and counselors use an intelligence test, they expect to get an intelligence quotient from it." Consequently, even a widely used test battery for multifactors is now sold to consumers with a key for a total score as well as for factor scores. Of special interest to statisticians is John W. Brent's paper "The Logic of and the Assumptions Underlying Differential Testing" in which he discusses multiple discriminant analysis and multiple regression techniques for guidance work. Here is a

field to which decision theorists could have important contributions, and in which their help is much needed.

The second portion of the *Proceedings* is largely for practitioners in the field of testing. Problems of explaining to consumers what tests can and cannot do, and problems of organization of testing programs, are discussed by various speakers. It may interest those outside of the testing profession to see with what skepticism the insiders hold the bulk of the tests manufactured commercially. It appears to be a sad fact that most tests being sold have poor reliabilities and validities. This situation is expected to continue for quite some time, since apparently it is difficult to teach the purchasers the methodological requirements of testing.

The fascinating problem of clinical vs. actuarial prediction is the topic of a panel discussion which constitutes the third section of the *Proceedings*. This was provoked by the recent book on this topic by Paul E. Meehl, who also served on the panel as final discussant. As far as this reviewer can make out, all the panelists agreed that actuarial prediction is superior to clinical prediction for the kinds of problems Meehl discussed in his book. The clinicians on the panel, however, point out that there are other important problems where the reverse is true, and indeed are the real reason for the clinical approach. This turns out to be Meehl's own position; he reports that he, as a clinician, wrote the book in order to dissuade clinicians from doing jobs which are better done by clerks, so that they should be more free to do jobs for which they are peculiarly suited. He himself is currently engaged in developing objective clinical procedures which he believes will be better than discriminant analysis.

This panel discussion reveals great need for new statistical methodology for studying universes of qualitative data. The future will undoubtedly see further developments along the lines of Zubin's pattern analysis, as well as other current qualitative theories not mentioned by the speakers. The panelists seem to give the impression that "actuarial" methodology is epitomized by discriminant analysis, and anything not using such an explicit framework is "clinical." But since all seem agreed that it is desirable to objectify clinical procedures as much as possible, it may be helpful to get rid of the notion that the specialized techniques known today are adequate for studying the qualitative universes of clinical phenomena, even from an "actuarial" point of view. This has been one of the most neglected fields of statistical methodology and the testing profession is only now appearing to wake up to the need for cultivating it.

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